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Guidelines for Displacement-based Design of Buildings and Bridges

Editor
Michael N. Fardis

Reviewer
Paolo Emilio Pinto

July, 2007
FOREWORD

Earthquake and landslide risk is a public safety issue that requires appropriate mitigation measures and means to protect citizens, property, infrastructure and the built cultural heritage. Mitigating this risk requires integrated and coordinated action that embraces a wide range of organisations and disciplines. For this reason, the LESSLOSS Integrated Project, funded by the European Commission under the auspices of its Sixth Framework Programme, is formulated by a large number of European Centres of excellence in earthquake and geotechnical engineering integrating in the traditional fields of engineers and earth scientists some expertise of social scientists, economists, urban planners and information technologists.

The LESSLOSS project addresses natural disasters, risk and impact assessment, natural hazard monitoring, mapping and management strategies, improved disaster preparedness and mitigation, development of advanced methods for risk assessment, methods of appraising environmental quality and relevant pre-normative research.

A major objective of the project is to describe current best practice and advance knowledge in each area investigated. Thus, LESSLOSS has produced, under the coordination of the Joint Research Centre, a series of Technical reports addressed to technical and scientific communities, national, regional and local public administrations, design offices, and civil protection agencies with the following titles:

Lessloss-2007/01: Landslides: Mapping, Monitoring, Modelling and Stabilization
Lessloss-2007/02: European Manual for in-situ Assessment of Important Existing Structures
Lessloss-2007/05: Guidelines for Displacement-based Design of Buildings and Bridges
Lessloss-2007/06: Application of Probabilistic Methods to Seismic Assessment of Existing Structures
Lessloss-2007/07: Earthquake Disaster Scenario Predictions and Loss Modelling for Urban Areas
Lessloss-2007/08: Prediction of Ground Motion and Loss Scenarios for Selected Infrastructure Systems in European Urban Environments
# LIST OF LESSLOSS-2007/05 CONTRIBUTORS

<table>
<thead>
<tr>
<th>Part</th>
<th>Section</th>
<th>Name:</th>
<th>Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2.1.1, 2.1.2</td>
<td>N. Ile, J.-M. Reynouard</td>
<td>INSA-LYON, France</td>
</tr>
<tr>
<td>I</td>
<td>2.1.3</td>
<td>S. Grange, P. Kotronis, J. Mazars</td>
<td>Institut National Polytechnique de Grenoble (INPG), France</td>
</tr>
<tr>
<td>I</td>
<td>2.1.4</td>
<td>R. Pinho, S. Antoniou, M. Lopez, H. Crowley</td>
<td>University of Pavia, Italy, SeismoSoft, Chalkida, Greece, ROSE School, Pavia, Italy, EUCENTRE, Pavia, Italy</td>
</tr>
<tr>
<td>I</td>
<td>2.2.1</td>
<td>D. Biskinis, M.N. Fardis</td>
<td>University of Patras, Greece</td>
</tr>
<tr>
<td>I</td>
<td>2.2.2</td>
<td>M.J.N. Priestley, C. Blandon, D. Grant</td>
<td>ROSE School, Pavia, Italy</td>
</tr>
<tr>
<td>I</td>
<td>3.1</td>
<td>D. Biskinis, M.N. Fardis</td>
<td>University of Patras, Greece</td>
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<tr>
<td>I</td>
<td>1.1</td>
<td>G. Ayala, C. Paulotto, F. Taucer</td>
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<td>I</td>
<td>1.2</td>
<td>V. Bardakis, M.N. Fardis, T.B. Panagiotakos</td>
<td>University of Patras, Greece, DENCO, Athens, Greece</td>
</tr>
<tr>
<td>I</td>
<td>2.1.1</td>
<td>T.B. Panagiotakos, B. Kolias</td>
<td>DENCO, Athens, Greece</td>
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<tr>
<td>I</td>
<td>2.1.2</td>
<td>R. Pinho, C. Casarotti</td>
<td>University of Pavia, Italy, ROSE School, Pavia, Italy</td>
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<tr>
<td>I</td>
<td>2.2.1</td>
<td>D. Biskinis, M.N. Fardis</td>
<td>University of Pavia, Italy</td>
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<tr>
<td>I</td>
<td>2.2.2</td>
<td>C. Paulotto, G. Ayala, F. Taucer</td>
<td>JRC, Ispra, Italy</td>
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<tr>
<td>I</td>
<td>3.1.1</td>
<td>D. Biskinis, M.N. Fardis</td>
<td>University of Patras, Greece</td>
</tr>
<tr>
<td>I</td>
<td>3.1.2</td>
<td>C. Paulotto, F. Taucer, G. Ayala</td>
<td>JRC, Ispra, Italy</td>
</tr>
<tr>
<td>I</td>
<td>3.2.1</td>
<td>T.B. Panagiotakos, B. Kolias</td>
<td>DENCO, Athens, Greece</td>
</tr>
<tr>
<td>I</td>
<td>3.2.2</td>
<td>C. Katsaras, T.B. Panagiotakos, B. Kolias</td>
<td>DENCO, Athens, Greece</td>
</tr>
<tr>
<td>I</td>
<td>3.2.3</td>
<td>G.M. Calvi, P. Ceresa, C. Casarotti, D. Bolognini, F. Auricchio</td>
<td>University of Pavia, Italy, ROSE School, Pavia, Italy, EUCENTRE, Pavia, Italy, University of Pavia, Italy</td>
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</tbody>
</table>
ABSTRACT

Displacement-based seismic design has now come of age, especially for buildings. This report attempts to contribute to it, by focusing on special subjects which are crucial for its further advancement. It is divided in two Parts: one for buildings and another for bridges. Both parts have a chapter on the estimation of displacement and deformation demands and another on component force and deformation capacities.

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# TABLE OF CONTENTS

FOREWORD.............................................................................................................................................. i

LIST OF LESSLOSS-2007/05 CONTRIBUTORS................................................................................ iii

ABSTRACT .................................................................................................................................................. v

TABLE OF CONTENTS............................................................................................................................. vii

Part I: Displacement-Based Design of Buildings................................................................................... 1

1. DISPLACEMENT-BASED DESIGN METHODOLOGIES FOR BUILDINGS................................. 3

2. ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BUILDINGS............. 7

## 2.1 Analysis methods for estimation of displacement and deformation demands in buildings

2.1.1 Evaluation of nonlinear analysis and modelling at various degrees of sophistication, on the basis of experimental results – Main model parameters affecting reliability of deformation predictions of nonlinear analysis................................................................. 7

2.1.2 Nonlinear dynamic versus linear analyses – static or modal – for irregular in plan buildings..................................................................................................................................................... 17

2.1.3 Simplified analysis of soil-structure interaction in 3D, including uplift ............................. 25

2.1.4 Adaptive pushover analysis for 2D irregular buildings in 3D.......................................... 37

## 2.2 Tools for estimation of displacement and deformation demands in buildings

2.2.1 Effective elastic stiffness of RC members for use in linear analyses emulating nonlinear ones ......................................................................................................................................................... 44

2.2.2 Ductility-dependent equivalent damping for displacement-based design (DDBD) ... 53
LESSLOSS – Risk Mitigation for Earthquakes and Landslides

3 ESTIMATION OF COMPONENT FORCE AND DEFORMATION CAPACITIES IN BUILDINGS

3.1 Acceptance and design criteria in terms of deformations, for RC members under uni- or bi-directional cyclic loading, at different performance levels

3.1.1 RC member deformation-based design criteria for Performance-based seismic design of buildings

3.1.2 Deformations of RC members at yielding

3.1.3 Flexure-controlled ultimate deformations under uniaxial loading

3.1.4 Flexure-controlled ultimate deformations under biaxial loading

3.1.5 Acceptable ultimate deformations for RC members under uni- or bi-directional cyclic loading, at different performance levels

3.1.6 Shear resistance in diagonal tension under inelastic cyclic deformations after flexural yielding

3.1.7 Shear resistance of walls or squat columns in diagonal compression under cyclic deformations

3.1.8 Acceptable shear resistance under inelastic cyclic deformations after flexural yielding, for different performance levels

Part II: Displacement-Based Design of Bridges

1 DISPLACEMENT-BASED DESIGN METHODOLOGIES FOR BRIDGES

1.1 Evaluation of iterative displacement-based design procedures for bridges

1.1.1 Introduction

1.1.2 State-of-the-art

1.1.3 Method based on the Substitute Structure

1.1.4 Method based on the Non-linear Capacity of the Structure

1.1.5 Application Examples

1.1.6 Concluding remarks
## Guidelines for Displacement-Based Design of Buildings and Bridges

1.2 Design of Bridge Piers Directly on the Basis of Displacement and Deformation Demands, Without Iterations with Analysis

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2.1</td>
<td>Proposed displacement-based design procedure</td>
<td>96</td>
</tr>
<tr>
<td>1.2.2</td>
<td>Nonlinear modelling of bridges with monolithic deck-pier connection</td>
<td>99</td>
</tr>
<tr>
<td>1.2.3</td>
<td>Nonlinear dynamic vs modal response spectrum analysis</td>
<td>102</td>
</tr>
<tr>
<td>1.2.4</td>
<td>Application of the proposed design procedure and evaluation of the design</td>
<td>108</td>
</tr>
</tbody>
</table>

2 Estimation of Displacement and Deformation Demands in Bridges

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Analysis Methods for Estimation of Displacement and Deformation Demands in Bridges</td>
<td>111</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Procedures for estimation of pier inelastic deformation demands via nonlinear analysis (static or dynamic) of the bridge</td>
<td>111</td>
</tr>
<tr>
<td>2.1.2</td>
<td>Displacement-based adaptive pushover analysis for bridges</td>
<td>122</td>
</tr>
<tr>
<td>2.2</td>
<td>Tools for Estimation of Displacement and Deformation Demands in Bridges</td>
<td>130</td>
</tr>
</tbody>
</table>

3 Estimation of Component Force and Deformation Capacities in Bridges

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Force and Deformation Capacity of Concrete Piers Under Cyclic Loading</td>
<td>145</td>
</tr>
<tr>
<td>3.1.1</td>
<td>Simple rules for the estimation of the flexure- or shear-controlled cyclic ultimate deformation of concrete piers, on the basis of test results</td>
<td>145</td>
</tr>
<tr>
<td>3.1.2</td>
<td>Models/procedures for estimation of ultimate deformations and shear capacity of concrete piers on the basis of the results of numerical analysis calibrated against tests</td>
<td>149</td>
</tr>
<tr>
<td>3.2</td>
<td>Displacement and Force Capacity of Isolators</td>
<td>168</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Evaluation of displacement capacity of isolators and of the associated overstrength – effect of exceedance of isolator displacement capacity on the bridge seismic response</td>
<td>168</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Displacement re-centring capacity of bridge isolation systems</td>
<td>180</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Effect of axial force variation on seismic response of bridges isolated with friction pendulum systems</td>
<td>191</td>
</tr>
</tbody>
</table>

References                                                                                   | 199  |
LIST OF SYMBOLS AND ABBREVIATIONS

215
Part I: Displacement-Based Design of Buildings
1. DISPLACEMENT-BASED DESIGN METHODOLOGIES FOR BUILDINGS

Structural displacements and member deformations do not enjoy a primary role in current force-based earthquake-resistant design. Their absolute magnitude is of interest only for aspects considered of secondary importance for seismic performance and safety: for the calculation of P-∆ effects, for the limitation of nonstructural damage through the control of interstorey drifts, for the control of pounding between adjacent structures, etc. In the main phases of current force-based design, namely that of member dimensioning for given strength demands and of member detailing, structural displacements and member deformations enter in an average sense and indirectly, through their ratio to the corresponding value at yield: through the displacement ductility ratios, global and local, which determine the global behaviour or force reduction factor, and the member detailing requirements, respectively. Recent years have seen, however, an increased interest in the absolute magnitude of displacements and deformations as the basis of seismic design. The main reason for this is the recent recognition that displacement- and deformation-, rather than strength-, demands and capacities, is what determines seismic performance and safety. The earthquake is a dynamic action, representing for a structure a demand to withstand certain displacements and deformations, but not specific forces. Ductility factors, although convenient for the determination of strength demands, are poor descriptors of deformation capacity, as the introduction of another, sometimes ill-defined variable, i.e. the yield displacement or deformation, often increases, instead of reducing uncertainty.

The concept of ‘displacement-based design’ (DBD) was introduced in the early 1990s as a logical and rational alternative to the current ‘force-based design’. In displacement-based procedures for the seismic design of new structures or the evaluation of existing ones, seismic displacements are the primary response variables for the design or the evaluation. This means that design or acceptance criteria and capacity-demand comparisons are expressed in terms of displacements rather than forces. Since their introduction in the early 1990s, displacement-based concepts have found their way more into seismic evaluation or assessment of existing structures, than in the design of new ones. For existing structures the application of displacement-based concepts is rather straightforward: the geometry of the structure and the reinforcement are known and simple or advanced analysis procedures can be employed for the estimation of inelastic displacement and deformation demands throughout the structure, to be compared with
member deformation capacities. Full application of displacement-based design to new structures is still facing difficulties. For example if the reinforcement of reinforced concrete (RC) members has not been determined yet, the distribution of a given global displacement demand to individual members is difficult. A second difficulty for RC structures is that, except in few special cases, direct procedures for reinforcement proportioning on the basis of given deformation demands have not been developed yet, to replace time-proven strength-based procedures for member proportioning. Recourse to iterations between member design and nonlinear analysis is often necessary to overcome the first difficulty. To bypass the second one, practically all DBD procedures proposed so far translate global displacement demands into a global strength demand, expressed in terms of a design base shear.

Nonlinear-static procedures of analysis (the so-called Pushover analysis methods) go hand-in-hand with displacement-based seismic design, as they are normally employed for the evaluation of a design produced by DBD. They also share with DBD common acceptance and evaluation criteria, namely the magnitude of inelastic member deformations. Nonlinear analysis, of the static (Pushover) or dynamic type, is also accepted by Eurocode 8 as a means for the design of new buildings without recourse to a global behaviour factor, q. In such an approach members are dimensioned/checked on the basis of acceptance criteria in terms of deformations, instead of forces. Nonetheless, the elaboration of such criteria is left for the National Annexes of Eurocode 8. A similar gap exists in Eurocode 8 regarding member (nonlinear) models to be used within the framework of nonlinear analysis, static or dynamic. An additional difficulty comes from the fact that common (nonlinear) member models require the geometry and the reinforcement of members to be known in detail a-priori, while such information is not available unless the structure has been fully designed. It seems, therefore, that although Eurocode 8 has opened the door for the use of nonlinear analysis, static or dynamic, for the direct seismic design of buildings, it has failed so far to provide to designer the tools they need to use this option. The reason of this failure is simply the lack of tools of this kind (member acceptance criteria in terms of deformations, simple and validated nonlinear member models, etc.) widely accepted within the international scientific and technical community.

Displacement-based design has to go beyond the methods that assume a single-degree-of-freedom representation of the structure. Such an assumption results in a severe restriction of the reliability of the estimated demands. At some risk of sacrificing simplicity, it is important to obtain a good estimate of the local displacement demands within the structure and to have an idea of how deformations are distributed. Therefore it is important to take higher-mode effects into consideration and to account for the sequence of element damage.
The concepts for displacement-based seismic design of new structures seem to have matured to the point that their implementation in codified seismic design seems feasible in the foreseeable future. Certain gaps need to be filled, though, before such an implementation. Some of these gaps are common with nonlinear analysis, static or dynamic, as noted above. More specific to DBD is the need for extension, elaboration and calibration of the methodology to buildings irregular in plan, which need to be analysed in three dimensions (3D) and their columns designed and detailed for strongly bi-directional deformation demands. As a matter of fact, lack of symmetry and uniformity in the plan layout of structural elements (plan “irregularity”) is quite common in the seismic regions of Europe, even in new buildings. Plan irregularity is considered to have considerably contributed – through the resulting torsional response around vertical axis – to heavy damages and collapses in past and recent earthquakes. Current seismic design codes for new buildings, including Eurocode 8, do not treat the problem of irregularity and torsional response sufficiently. The relevant clauses are based on elastic considerations of simple models or are empirical. Moreover, some codes, especially US ones confuse the problem of torsional response of irregular structures by assuming independence between strength and stiffness of structural elements and using member displacement ductility factors as the underlying criterion for member design. A DBD approach takes realistically into account the coupling between member strength and stiffness (by using yield deformations that depend on member dimensions and not on member strength) and places the emphasis on absolute member deformations as the criterion. So, it is intrinsically better suited than forced-based approaches to tackle the problem of torsion of irregular structures. The primary question to be resolved, then, is the calculation of member inelastic displacement demands in 3D in the presence of torsion, in the framework of DBD.

In closing this Introduction, it is emphasised that displacement-based concepts represent the most rational and appropriate basis for performance based seismic design of structures During the last decade, research activities in earthquake engineering have witnessed an increasing pressure from owners, insurance companies, politicians and engineers to re-evaluate and improve the state of practice of seismic design to meet the challenge of reducing life losses and the huge economic impact attributed to recent earthquakes, which by no means could be considered as unusual or rare. As a result of this pressure, different research groups have reinitiated the investigations on the concepts and procedures for the performance based seismic evaluation and design of structures.
2. ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BUILDINGS

2.1 ANALYSIS METHODS FOR ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BUILDINGS

2.1.1 Evaluation of nonlinear analysis and modelling at various degrees of sophistication, on the basis of experimental results – Main model parameters affecting reliability of deformation predictions of nonlinear analysis

2.1.1.1 Introduction

Despite the fact that the advantages of using inelastic analysis for design purposes have been recognized since the eighties, a gap still exists in Eurocode 8 regarding member nonlinear models to be used within the framework of nonlinear analysis. For reinforced concrete slender elements, the most promising model among the member-type models of the flexural behaviour of reinforced concrete members is the fibre or semi-local approach. In a fibre model the member is discretized both longitudinally, into segments represented by discrete slices and at the cross-sectional level, into finite regions. In contrast with the member-type models which try to capture the overall behaviour through semi-empirical hysteresis relations between moment and curvature or moment and plastic hinge rotation, fibre models use directly realistic $\sigma - \varepsilon$ laws of individual material components (including effects such as tension-stiffening of reinforcement, confinement of the concrete, etc.). Therefore, the fibre or semi-local approach can account for the details of the geometry of the cross-section and of the distribution of the reinforcement, and takes properly into account the interaction between axial force and one directional bending, between the two directions of bending and between the two directions of bending and the axial force.

In members with low shear span ratio, shear deformations are almost as important as flexural deformations. For large amplitude reversed loading, these members exhibit significant degradation of stiffness and strength with cycling. Actually, after a few load cycles, shear deformations may exceed the flexural ones, leading eventually to a shear failure. When non-linear response is of interest, the behaviour of this type of members is strongly influenced by the interaction between axial force, flexure and shear, and the macro type of modelling, as well as the fibre approach, will have some inherent
difficulties in accurately reproducing all these phenomena. Clearly, the adoption of a local approach (with modelling of members at a point-by-point basis), based upon cyclic behaviour laws of the constitutive materials, seems more appropriate in this case. With this kind of models, since the interaction between axial force, flexure and shear is directly taken into account, it is expected to obtain more accurate predictions of the seismic response for local inelastic deformations, degradation of strength and stiffness of the member and energy dissipation through hysteresis.

Using the fibre and local approach type of modelling, a series of nonlinear analyses have been performed for RC frame and wall members as well for RC structures. In a first phase, using cyclic test results on RC members, two member types were selected for comparative evaluation of the reliability of the above mentioned nonlinear models. The first member type is a reinforced square concrete column built as a cantilever into a heavily reinforced foundation, tested by Bousias et al. [1995] under biaxial bending with constant axial load. The second member type is represented by a low-rise reinforced concrete shear wall which was tested under in-plane pseudodynamic loading at the ELSA reaction wall of JRC, in pure shear. In a second phase, 3D response and torsion of RC irregular frame and wall structures is simulated and numerical response is compared with the measured response. The first example is an irregular RC frame structure (the SPEAR structure), tested under bidirectional input at the ELSA reaction wall of JRC. The second one is a RC asymmetric bearing wall structure (CAMUS 2000-2) tested on the CEA shaking table in Saclay.

2.1.1.2 Evaluation of nonlinear analysis models

Fibre type of modelling

The fibre type of modelling taking advantage of the simplified kinematic hypothesis of the Euler-Bernoulli or Timoshenko theories offers a reliable and practical solution for the nonlinear analysis of reinforced concrete elements. For the following simulations, the nonlinear Timoshenko beam element with fibre type assumptions at the section level and implemented in the finite element code CAST3M [Guedes et al., 1994] has been adopted. This modelling approach is based on the Timoshenko beam theory, assuming that plane sections remain plane after deformation but not necessarily normal to the beam axial axis. The fibre modelling considered in the following applications is a simplified one, because it accurately accounts only for the interaction between normal force and bending, the calculation of shear deformation and stiffness being uncoupled from the flexural mode.

As can be seen in Figure 2.1, fibre modelling is based on the geometrical description of the beam section in fibres. Each fibre supports a uniaxial law representative of concrete or steel behaviour. Figure 2.2 and Figure 2.3 show the laws used in the present study.
respectively for concrete and for steel. Concrete behaviour is represented by a parabolic curve up to the peak stress point followed by a straight line in the softening regime. Cyclic behaviour takes into account the decrease in material stiffness and crack closing phenomena. The relation between tensile stress and crack strain has a linear form and is determined by two parameters, tensile strength $f_t$ and crack opening strain $\varepsilon_{om}$. The presence of discrete concrete cracking, of the contribution of concrete in tension between the cracks, and of the non-linear bond-slip relation can be taken into account, in an average sense, by appropriately modifying the $\sigma - \varepsilon$ relation of concrete after cracking. For the present study, a simplified assumption has been adopted. It is based on the suggestion of Barzegar and Schnobrich [1986], namely that when the reinforcing steel intersects the cracks at right angles an appropriate value for $\varepsilon_{om}$ is the yield strain of the longitudinal reinforcement. Confinement is taken into account by the modification of the plain concrete curve and by including an additional plateau phase. The classical approach to model the steel behaviour is to adopt an isotropic kinematic law with a linear hardening. However, more realistic laws have been proposed in the past to better describe the steel behaviour under cyclic loading by adequately describing some fundamental phenomena like the Bauschinger and buckling effects. Therefore, in this study a more advanced model is adopted - the modified Menegotto and Pinto [1973] model - to represent the cyclic behaviour of steel.

![Figure 2.1: Nonlinear fibre beam model](image)

Using the fibre approach, the cyclic behaviour of a reinforced square concrete column, which was tested by Bousias under biaxial bending with constant axial load was simulated. Then the same model was employed to study the seismic response of a RC irregular frame structure (the SPEAR structure) which was tested under bidirectional input at the ELSA reaction wall of JRC. The column specimen tested by Boustas et al [1995] and the SPEAR
structure are represented in Figures 2.4 and 2.5. On the same Figures, the finite element models used in the numerical analysis are also presented.

Figure 2.2 Uniaxial constitutive law for concrete

Figure 2.3: Menegotto-Pinto [1973] uniaxial law with Bauschinger effect and buckling for steel
In order to identify the main model parameters affecting the reliability of the predictions of nonlinear analysis several modelling assumptions were considered through parametric analyses. The comparison between numerical and experimental results indicated that fibre models are well suited (provided certain assumptions are fulfilled) for the analysis of slender elements and more generally for the analysis of three-dimensional frames. Nonlinear analysis results were generally consistent with the experimental time-history displacements and were able to predict the global failure mechanism of the 3-D frame structure with reasonable accuracy.

The fundamental parameters that may enhance or reduce the accuracy of the predicted response are briefly discussed below:
• For describing the cyclic behaviour of the reinforcement, as compared to the classic approach based on an isotropic hardening law, the use of the modified Menegotto-Pinto model which takes into account the Bauschinger effect and when necessary, the buckling of reinforcing bars, provides a better description of the energy dissipation through hysteresis.

• It was also found that it is important to take into account in the model the confinement effect of concrete through confinement parameters, depending on the section characteristics. If this effect is not properly taken into account, simulation of seismic response may lead to unrealistic member strength results.

• Other concrete model parameters such as the tensile strength $\sigma_t$ and the ratio $r = \varepsilon_{cm}/\varepsilon_u$ have generally very limited influence on the numerical results, when cyclic response is of interest.

• If the beams of a frame are substantially stronger than its columns, the structure will tend to behave inelastically with nonlinear excursions in the vertical elements, leading to a soft-storey mechanism, with no energy dissipation contributed by the beams. Moreover, if smooth bars are used in construction, bond-slip will cause the rotation at column ends and will contribute to the storey drift. If bond slip is not explicitly integrated in the fibre model, centreline dimensions of elements rather than rigid links are more appropriate to model the overall behaviour of a RC frame structure with smooth bars.

Material level modelling

In a material level modelling, a reinforced concrete structure is discretized into a large number of finite elements, with different elements used for the concrete and for the reinforcing steel and possibly for their interaction through bond. Such modelling allows, in principle, representation in the analysis of the necessary geometric characteristics, and construction details of a RC structure, as well as of the boundary conditions. In the last years, considerable progress has been made in the field of constitutive modelling of plain or reinforced concrete under generalized multiaxial loading, including reversals. Despite this progress, the computational and memory requirements of such a refined approach have restricted its application to the analysis of the response of individual members, especially shear walls. Presently, however, efforts to extend the application of such a detailed modelling to the seismic analysis of RC wall structures at reduced scale have been successful in reproducing the experimentally observed behaviour. A numerically convenient way to achieve this is by using a smeared crack approach for crack representation and by using uncomplicated cyclic behaviour laws for concrete. In the following, the basic features of a biaxial concrete model, representing a good compromise between simplicity and accuracy and providing acceptable representation of the cyclic
The concrete constituitive model [Merabet and Reynouard, 1999], adopts the concept of a smeared crack approach with a possible double cracking only at 90°. It is based upon the plasticity theory for uncracked concrete with isotropic hardening and associated flow rule. Two distinct criteria describe the failure surface: Nadai in compression and bi-compression and Rankine in tension. Hardening is isotropic and an associated flow rule is used. When the ultimate surface is reached in tension, a crack is created perpendicular to the principal direction of maximum tensile stress, and its orientation is considered subsequently as fixed. Each direction is then processed independently by a cyclic uniaxial law, and the stress tensor in the local co-ordinate system defined by the direction of the cracks is completed by the shear stress, elastically calculated with a reduced shear modulus $\mu G$, (with $0 < \mu < 1$, and $\mu$ being a function of the crack opening strain) to account for the effect of interface shear transfer:

\[
\mu = \begin{cases} 
0.0 & \text{if } \varepsilon_{\text{cr}} - \varepsilon_{\text{res}} - \varepsilon_{\text{tm}} \leq 2 \varepsilon_{\text{tm}} \\
0 & \text{if } \varepsilon_{\text{cr}} - \varepsilon_{\text{res}} - \varepsilon_{\text{tm}} \geq 2 \varepsilon_{\text{tm}} \\
0 \text{ and } \sigma_{12} = 0 & \text{if } \varepsilon_{\text{cr}} - \varepsilon_{\text{res}} - \varepsilon_{\text{tm}} \geq 4 \varepsilon_{\text{tm}}
\end{cases}
\]  

(2.1) (2.2) (2.3)

where:

$\varepsilon_{\text{cr}}$ - total strain

$\varepsilon_{\text{res}}$ - residual strain after unloading in compression

$\varepsilon_{\text{tm}}$ - crack opening strain

$\sigma_{12}$ - shear stress
The uniaxial law implemented in each direction allows accounting for the main phenomena observed during a loading comprising a small number of cycles. According to the constitutive cyclic law for concrete, as soon as a crack starts to close, the concrete develops some compression, due to the imperfect matching of the crack surfaces. Furthermore the model considers damage of the elastic modulus and of the tensile resistance as the inelastic compressive strains increase. The behaviour of a point initially under tension, which completely cracks prior to undergoing reverse loading in compression, is illustrated in Figure 2.6. Similar laws describe the case of an initially compressed point or that of a point which has not totally cracked under reverse loading. The model has been described in detail by [Ile and Reynouard, 2000, Ile et al., 2002].

The finite element model and the assigned non-linear material properties can be improved by modelling the bond-slip interaction between the steel bars and concrete through the use of special contact elements, interposed between steel and concrete elements. If such elements are used in conjunction with the smeared modelling of cracks, they can only reflect the trend of the variation of steel and concrete stresses along the bar, to which such contact elements are connected. Local variation between cracks is unaccounted, since location of individual cracks cannot be accurately predicted by the smeared crack model. Moreover, for smeared cracking it is computationally more efficient to assume perfect bond between steel and concrete and account implicitly for the effect on bond-slip on the average stresses and strains in concrete, through appropriate modification of the properties of concrete. This latter approach was adopted in this study. So, tension
stiffening was superimposed to the tension softening following a simple approach proposed by Feenstra and de Borst [1995].

Using the previously described concrete model, the cyclic behaviour of a low-rise RC shear wall which was tested under in-plane pseudodynamic loading at the ELSA reaction wall of JRC in pure shear, was simulated. Then the same model was employed to study the 3D torsional response of a RC irregular wall structure (the CAMUS 2000-2 structure) which was tested under unidirectional input on the shaking table of CEA in Saclay. The numerical analyses have been performed using the general-purpose finite element program CAST3M. The wall specimen tested at the ELSA reaction wall and the CAMUS 2000-2 structure are represented in Figures 2.7 and 2.8. On the same Figures, the finite element models used in the numerical analysis are also represented.

![Figure 2.7: Shear wall specimen and finite element model](image)
Using nonlinear time-history analysis, member inelastic displacement and force demands were calculated and then compared to available experimental results. Nonlinear analysis results were generally consistent with the experimental time-history displacements and were able to predict the global failure mechanism of the structure with reasonable accuracy. At a more local level, the essential features of the observed structural response were also well reproduced. As was experimentally observed, the model predicted failure of the shear wall specimen by compression failure of the concrete at the base of the wall, while in the case of the irregular CAMUS2000-2 structure, failure was predicted in the long wall by the opening of a wide horizontal crack and rupture of the reinforcement at the first level.

Despite the good correlation between the predicted and the measured response, global response variables as force and displacement being reasonably close to the experimental one, energy dissipation through repeated cycles was underestimated by the existing model. Apart from the friction between the structure and the environment, it could be assumed that damping is still of hysteretic origin, coming from the friction between the faces of the cracks during closing. Hence, at the expense of adding some supplementary hysteretic loops in the model, a better description of the hysteretic energy could be obtained.

The model represents however a good compromise between simplicity and accuracy, and provides a correct representation of the cyclic inelastic behaviour of reinforced concrete under cyclic loading. As a matter of fact, underestimating the energy dissipation leads to conservatism in estimating forces and displacements and hysteretic damage under cyclic loading seems to be less important as far as the force-ductility demand is concerned.

The principal advantage of this kind of models lies in the fact that the mechanical
properties of each constituent are based on the actual local behaviour of materials, and due to this the interaction between axial force, flexure and shear is directly taken into account. Since nowadays computer power is continuously increasing, the major drawback of the refined type of modelling, namely the extensive computational effort, seems to become less important in the near future.

2.1.2 Nonlinear dynamic versus linear analyses – static or modal – for irregular in plan buildings

2.1.2.1 Introduction

It is now generally accepted that a DBD approach can take realistically into account the coupling between member strength and stiffness (by using a yield deformation that depends on dimensions of the member and not its strength). Therefore, it is better suited than force-based design approaches to tackle the problem of 3D response and torsion of irregular structures. However, in order to obtain reliable estimations of member inelastic demands, DBD has to go beyond the methods that assume a single-degree-of freedom representation of the structure, because such an assumption results in a severe restriction of the reliability of the estimated demands. At some risk of sacrificing simplicity, it is important to obtain a good estimate of the local displacement demands within the structure and to have an idea how deformations are distributed. Therefore it is important to take higher-mode effects into consideration and to account for the sequence of element damage. It is necessary then, to analyse the accuracy of the estimation of member inelastic seismic demands, using appropriate analysis methods. To tackle the problem of 3D response and torsion of irregular structures two analysis methods were considered in this study:

• The time-history analysis method based on two different modelling approaches:
  – fibre type modelling for the 3D behaviour of irregular frame structures, and
  – 3D thin shell type of modelling for the 3D behaviour of irregular shear wall structures

• The modal response spectrum analysis, with member effective stiffness equal to:
  – the “empirical effective stiffness” proposed in Section 2.2.1.4, and
  – 50% of that corresponding to the initial uncracked stiffness

The following two structures were selected for comparative evaluation of the different
numerical approaches:

An irregular frame structure tested under bidirectional seismic input at the ELSA reaction wall of JRC. Two different configurations were considered:

- the original “as built” specimen and the structure retrofitted with RC jacketing.
- An asymmetric reinforced concrete bearing wall specimen tested on the shaking of CEA in Saclay.

2.1.2.2 Plan-wise irregular RC frame structure (the SPEAR structure)

It was stated in Section 2.1.1 that nonlinear time-history analysis results were generally consistent with the experimental time-history displacements and were able to predict the global failure mechanism and torsional behaviour of the SPEAR structure with reasonable accuracy.

As alternative to the non-linear time-history analysis, the linear time-history and the modal response spectrum analysis, which are more familiar to the practicing engineer are used here to assess the behaviour of the irregular SPEAR structure. In fact, the modal response spectrum analysis is the reference method for determining the seismic effects in Eurocode 8, Part 1 [CEN, 2004]. In principle, it can provide improved representation of higher mode effects and enables torsional response to be included when determining displacement demands. A fundamental requirement for this very convenient approximation is to use a realistic estimate of the elastic cracked stiffness of concrete members at yielding. To reflect this condition, Eurocode 8 requires that the stiffness of concrete members corresponds to the initiation of yielding of the reinforcement (secant stiffness to the yield-point). Unless a more accurate modelling of the cracked member is adopted, Eurocode 8 and US codes allow taking that stiffness equal to 50% of the corresponding stiffness of the uncracked member, $E_c I_c$, neglecting the presence of reinforcement.

It is to be noted that linear modal analysis is only an approximation of the true inelastic response. However, it is a very convenient approximation for the case of asymmetric structures where torsional response is important and may conduct to a realistic estimation of the chord rotation demands throughout the structure if:

the ductility demands are not concentrated to a part of the structure, but rather uniformly distributed throughout it, and

- realistic values of the elastic cracked stiffness of concrete members (a representative value of the member secant stiffness to yielding) are used in analysis.
For existing structures, as the geometry and the reinforcement of the structure are known, the secant stiffness to yielding of RC members may be readily estimated. This may be calculated assuming purely flexural behaviour, from section moment-curvature relations and integration along the member length. For new structures, one of the difficulties is that the secant stiffness to yielding can not be reliably estimated before the seismic response analysis, because at that stage member reinforcement has not been determined yet. Hence, recourse to iterations between member design and analysis of seismic response is generally required. For the case under study, despite the fact that the geometry and the reinforcement of the test specimen were known, to better assess the performance of the multimodal analysis, it was preferred to adopt a secant to yielding stiffness based on geometric and other characteristic of the members, known before dimensioning and detailing of its reinforcement. The expression used here is the "empirical effective stiffness" proposed in Section 2.2.1.4.

The response of 10 modes was taken into account, as the sum of the effective modal masses for the first 10 modes amounts to about 99% of the total mass of the structure in each direction. The damping for the elastic model was defined to have 5% damping in the first ten natural modes of the test specimen. In the case of the modal response spectrum analysis, the “Complete Quadratic Combination” procedure was used for the combination of modal responses. The results of the time-history multimodal and that of the modal response spectrum analysis were almost identical. Therefore only the results corresponding to the time-history multimodal analysis are compared with the nonlinear time-history analysis results. A comparison of the results obtained from the application of the modal analysis procedure with those derived from the time-history nonlinear analysis, in terms of interstorey drifts and rotations is provided in Figure 2.9. It can be observed...
that modal analysis generally overestimates interstorey drifts in both directions and largely underestimates interstorey rotations. However, the distribution of interstorey drifts over the height of the structure provided by the modal analysis is consistent with that obtained from nonlinear analysis. Therefore, modal analysis can provide a relatively good description of the global failure mechanism of the structure.

Peak inelastic chord rotations computed at member ends by the time-history fibre model were divided by the corresponding elastic values from modal analysis using the 5% damped elastic spectrum. As an example, Table 2.1 presents means and upper characteristic values of this ratio (called “bias factor”) for chord rotations, for the “as built” structure. From this table it can be seen that the inelastic-to-elastic chord rotation ratio depends on the type of element (beam or column), its elevation in the structure and to a certain extent, especially for columns, on the intensity of ground motion. If one wants to refer to the highest level of applied input motion - PGA = 0.20 g, (which actually caused much more damage to the specimen than the 0.15 PGA motion), mean and 95%-fractile representative values for the “as built” structure are 1.06 and 1.91 for beams, and roughly 1.0 and 1.40 for columns. This suggests that, for the structure under study, accurate seismic response and torsional effects can be obtained without significantly increasing the uncertainty resulting from the approximate modal analysis, by applying corrective factors, for instance, those presented in Table 2.1.

Table 2.1: Mean and 95%-fractile of inelastic-to-elastic chord rotation for the “as built” structure (PGA = 0.15 and 0.20 g)

<table>
<thead>
<tr>
<th></th>
<th>Beam chord rotation</th>
<th></th>
<th>Column chord rotation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>95%-fractile</td>
<td>Mean</td>
<td>95%-fractile</td>
</tr>
<tr>
<td><strong>PGA = 0.15 g</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st level</td>
<td>1.06</td>
<td>1.76</td>
<td>0.70</td>
<td>0.92</td>
</tr>
<tr>
<td>2nd level</td>
<td>1.10</td>
<td>2.26</td>
<td>0.90</td>
<td>1.31</td>
</tr>
<tr>
<td>3rd level</td>
<td>1.10</td>
<td>1.99</td>
<td>0.70</td>
<td>1.0</td>
</tr>
<tr>
<td>Mean</td>
<td>1.09</td>
<td>2.0</td>
<td>0.77</td>
<td>1.07</td>
</tr>
<tr>
<td><strong>PGA = 0.20 g</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st level</td>
<td>0.96</td>
<td>1.68</td>
<td>0.66</td>
<td>0.96</td>
</tr>
<tr>
<td>2nd level</td>
<td>1.08</td>
<td>1.74</td>
<td>1.18</td>
<td>1.80</td>
</tr>
<tr>
<td>3rd level</td>
<td>1.15</td>
<td>2.31</td>
<td>1.13</td>
<td>1.43</td>
</tr>
<tr>
<td>Mean</td>
<td>1.06</td>
<td>1.91</td>
<td>0.99</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Figure 2.10 compares the analysis results obtained by using the stiffness of concrete members equal to 50% of the corresponding stiffness of the uncracked member, with test results. It can be observed that the use of the 50% of the uncracked gross section stiffness is largely conservative in terms of interstorey drifts. This is not the case, when a more realistic estimate of the effective elastic stiffness of concrete members is used (see Figure 2.10). Therefore, it can be concluded that if member stiffness is taken to the default value of 50% of the uncracked gross section stiffness, which is generally
recommended for force– and strength-based seismic design, overall and local seismic
deformation demands are seriously underestimated.

It is noted that the model structure studied here represents old structures without
significant engineered earthquake resistance, which typically do not satisfy the strong-
column/weak rule of capacity design. The test as well as the nonlinear analysis results
exhibit plastic hinges only in the columns and indicate that there is a strong tendency for
concentration of inelasticity in a single soft storey, namely the second one.

As for the modal analysis, it can provide on average sufficiently accurate global results,
but fails to capture accurately the detailed local behaviour. Therefore, for such cases, the
more cumbersome dynamic time-history analysis seems more appropriate for the
estimation of chord rotation demands and interstorey drifts. Nevertheless, for asymmetric
new RC structures, which typically satisfy deliberately the strong-column/weak beam rule
of capacity design, which have a more uniform distribution of inelasticity, it is expected
that the linear multimodal analysis can be used with sufficient accuracy. In these cases, it
is expected that the equal displacement rule will apply better at the level of member
deformation, provided that realistic values of the secant rigidity of the RC members at
yielding are used.

These conclusions are preliminary as they are based on a very particular structure. More
work is needed to consider more building configurations with other torsional-to-lateral
frequency ratios and the impact of the strong-column/weak beam rule of capacity design.
2.1.2.3 Plan-wise irregular RC wall structure (the CAMUS 2000-2 structure)

Two simplified methods are used in this section to assess the behaviour of the irregular CAMUS 2000-2 shear wall structure: the time history modal analysis and the modal response spectrum analysis. All the seismic signals applied to the specimens were considered. The predicted maximum top displacements are then compared with the experimental and nonlinear time-history analysis results. The effective stiffness of the two walls is calculated following the “empirical effective stiffness” proposed in Section 2.2.1.4, while the stiffness of the other structural members (slabs and bracing system) is taken equal to that corresponding to the initial uncracked elastic stiffness. For comparison purposes, in addition to analyses with the “empirical effective stiffness”, also one analysis with the effective stiffness of concrete equal to 50% of that corresponding to the initial uncracked stiffness was performed for the highest applied seismic signal.

The analysis was performed with the spatial model presented in Section 2.1.1.2. The response of 11 modes were taken into account, since the sum of the effective modal masses for the first 11 modes is greater than 90% of the total mass of the structure in each direction. The damping for the elastic model was defined as 5% in the first 11 natural modes of the test specimen.

The average top relative displacement and the top storey rotation at the centre of mass derived from the application of the modal analysis procedures are compared to the experimental and nonlinear results in Figures 2.11 and 2.12. It can be observed that modal response spectrum analysis using the “empirical effective stiffness” generally overestimates displacements and rotations. Up to 0.80g PGA level, experimental top displacements and rotations are better reproduced by the nonlinear analysis, while all the methods underestimate the top relative displacement for the last applied seismic signal. It is to be reminded that for the last seismic input (PGA = 1.12g) failure was obtained in the long wall by rupture of the reinforcement at the bottom of the wall, while the short wall did suffer less damage, as it is much more flexible. At this stage, the behaviour of the walls is asymmetric and redistribution of forces due to the evolution of stiffness of the walls is completely nonlinear and cannot be adequately represented in a standard linear elastic model. Therefore, the observed discrepancies seem to be acceptable. Moreover, it is to be noted that the difference between predicted and experimental top displacement is not very important: about 7% for the nonlinear analysis and 10-12% for the modal analysis methods.

Peak inelastic relative displacement demands at the top of the structure from nonlinear analysis were divided by the corresponding values from the modal spectrum analysis using the 5%-damped elastic spectrum. From Table 2.2 it can be observed that the inelastic-to-elastic displacement ratio seems relatively insensitive to the details of the
structural configuration, because the ratio values for the two walls are not too much separated from each other. However, this ratio depends on the PGA level of the seismic input. For PGA level lower than that corresponding to the design earthquake modal analysis may overestimate displacements by as much as 30-50%, but very good agreement with the nonlinear analysis results is obtained for the design earthquake (PGA = 1.12g). This is a confirmation of the fact that the “empirical effective stiffness” proposed in Section 2.2.1.4 reproduces well the secant member stiffness to yielding and so, good approximation of the displacement demands are obtained from a linear modal analysis.

Figure 2.11: Maximum average top relative displacement from analysis and experiment.
Figure 2.12: Maximum top storey rotation from analysis and experiment.

Table 2.2: Inelastic–to-elastic top storey displacement for the CAMUS 2000-2 structure

<table>
<thead>
<tr>
<th></th>
<th>PGA = 0.17g</th>
<th>PGA = 0.40g</th>
<th>PGA = 0.80g</th>
<th>PGA = 1.12g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short wall relative top displacement</td>
<td>0.47</td>
<td>0.43</td>
<td>0.73</td>
<td>0.97</td>
</tr>
<tr>
<td>Long wall relative top displacement</td>
<td>0.36</td>
<td>0.41</td>
<td>0.78</td>
<td>1.09</td>
</tr>
<tr>
<td>Average relative top displacement</td>
<td>0.42</td>
<td>0.42</td>
<td>0.75</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Figures 2.11 and 2.12 also show the analysis results obtained by using the stiffness of wall members equal to 50% of the corresponding stiffness of the uncracked member. It can be observed that the use of the 50% of the uncracked gross section stiffness leads to very unrealistic predicted response values, i.e. the top relative displacement is less than 50% of the measured displacement. As in the previously studied irregular frame structure, if member stiffness in this irregular wall structure is taken equal to 50% of the uncracked gross section stiffness, overall deformation demands are seriously underestimated.

In conclusion, the modal spectrum analysis is a viable alternative to nonlinear analysis, in case when 3D and torsional response is of interest. It can provide on the average conservative global results, even if it fails to capture accurately the detailed local behaviour. The main condition for this very convenient approximation is to use realistic values of the elastic cracked stiffness of concrete members (a representative value of the member secant stiffness to yielding). It was found that estimating the secant to yielding stiffness of RC members with the “empirical effective stiffness” proposed in Section
2.2.1.4 provides consistent global results, despite the fact that re-distribution of forces due to the changing stiffness of RC members, is highly nonlinear and cannot be accurately represented in a standard linear elastic model.

Finally, the importance of using a realistic estimate of the elastic cracked stiffness was highlighted by performing alternative modal spectrum analyses, the stiffness of concrete members being equal to 50% of the corresponding stiffness of the uncracked member. It was found that if member stiffness is taken to the default value of 50% of the uncracked gross section stiffness, which is generally recommended for force- and strength-based seismic design, overall deformation demands are seriously underestimated.

2.1.3 Simplified analysis of soil-structure interaction in 3D, including uplift

2.1.3.1 Description of the SSI Macro-Element

Introduction

In structural engineering, Soil Structure Interaction (SSI) is an important phenomenon that has to be taken into account. Experimental results on the CAMUS IV structure [Combescure et al., 2000, CAFEEL-ECOEST/ICONS, 2001] showed that non-linearities at the soil level (plasticity) and between the soil and the foundation (uplift of the foundation) result often to an isolation of the structure and thus to a reduction of the forces and the moments developed at its base during an earthquake. Maximum values of stresses are limited because of larger energy dissipation but more important displacements are generated at the top.

In order to study the SSI, several methods exist: the macro element approach consists in condensing all non-linearities into a finite domain and works with generalised variables (forces and displacements) that allow simulating in a simplified way the behaviour of shallow foundations. Several 2D macro elements exist in the literature: [Nova et al., 1991, Cassidy et al., 2002, Crémer, 2001, Crémer et al., 2001, Crémer et al., 2002]. The 2D macro element developed by Crémer can be used for static/cyclic but also dynamic loading (i.e. earthquake) applied in the horizontal direction, considering the plasticity of the soil and the rocking and uplift of the foundation.

Inspired by that work, a new 3D macro element is developed hereafter. The goal is to compute the 3D behaviour of a circular shallow and rigid foundation lying on an infinite space submitted to a static or a dynamic loading. In the current version the macro element takes into account the plasticity of the soil and the rocking of the structure. It is implemented into FedelasLab, a finite element Matlab toolbox developed by [Filippou et al., 2004].
After the mathematical description of the macro element, numerical results are compared with experimental tests under monotonic static [Gottardi et al., 1999], and dynamic [NEES7 story, 2006] loadings to show the good performance of the approach.

Shape of the foundation and associated kinematic variables

To simplify the problem, the foundation studied hereafter is considered circular (Figure 2.13). Because of the axisymmetry, the horizontal loads in the directions $x$ and $y$ are therefore computed in a similar way. Furthermore, it is easier to reproduce the interaction between horizontal forces and moments. Being a macro element, the foundation is supposed infinitely rigid and all non-linearities are condensed in a representative point: its centre. Within that framework it is appropriate to work with generalized (global) variables: the vertical force $V$, horizontal forces $H_x, H_y$ and moments $M_x, M_y$ but also the corresponding displacements: vertical settlement $u_z$, horizontal displacements $u_x, u_y$, and rotations $\theta_x, \theta_y$. Torque moment ($M_z$) is not taken into account by the model.

Mathematical description of the 3D macro element

The 3D macro element presented briefly hereafter takes into account the plasticity of the soil and the rocking of the structure. The classical partition of the total displacement $\vec{u}$ into an elastic part $\vec{u}^{el}$ and a plastic part $\vec{u}^{pl}$ ($\vec{u} = \vec{u}^{el} + \vec{u}^{pl}$) is assumed.

![Figure 2.13: Shape of the foundation and generalized variables: (a) forces and (b) displacements.](image-url)
Elastic behaviour

For a circular foundation, the stiffness corresponding to both horizontal displacements is the same. The same stands for rotations. The constitutive law can be written as: $$\vec{F} = K^{el} \left[ \vec{u} - \vec{u}^{nl} \right]$$ where the displacement and force vectors are dimensionless according to Eq. (2.4) ($D$ is the diameter and $q_{max}$ the ultimate bearing capacity of the foundation):

$$\vec{F} = \begin{pmatrix} V' \\ H'_{xx} \\ M'_{xx} \end{pmatrix} = \frac{1}{\pi D^2} q_{max} \begin{pmatrix} V \\ H_{xx} \\ M_{xx} \end{pmatrix} D, \text{ and } \vec{u} = \begin{pmatrix} u'_{x} \\ u'_{y} \\ \theta' \end{pmatrix} = \frac{1}{D} \begin{pmatrix} u_{x} \\ u_{y} \\ \theta \end{pmatrix}$$ \hspace{1cm} (2.4)

Using the dimensionless notation, a diagonal dimensionless stiffness matrix is found with the following terms in the diagonal ($S$ being the surface of the foundation):

$$K^{el}_{xx} = \frac{K^{el}_{xx} D}{S q_{max}}, \hspace{0.5cm} K^{el}_{yy} = \frac{K^{el}_{yy} D}{S q_{max}}, \text{ and } K^{el}_{\theta\theta} = \frac{K^{el}_{\theta\theta} D}{S q_{max}}$$ \hspace{1cm} (2.5)

The elastic stiffness matrix is calculated using the real part of the static impedances of the foundation [Gazetas, 1991].

Plastic behaviour

(a) Failure criterion. The failure criterion is defined for an overturning mechanism with uplift. It comes from the works of [Pecker, 1997] and it has been used already in the 2D macro element of [Crémer, 2001]. This criterion was initially developed for a 2D shallow strip and rigid foundation lying on a half space of homogeneous cohesion. However, [Gottardi et al., 1999] showed that the shapes of the load and failure surfaces for a circular footing are similar. Considering this and thanks to the symmetry of revolution, the adaptation in 3D is straightforward. It consists in adding 2 terms in relation with the horizontal force $H'_{x}$ and the moment $M'_{y}$ to obtain a 5D surface:
With the coefficients:

- \( a, b \) defining the size of the surface in the planes \((H' - M')\)
- \( c, d, e \) and \( f \) defining the parabolic shape of the surface in the planes \((V' - M')\) and \((V' - H')\)

These parameters can be fitted to different experimental results found in the literature (see for example the numerical simulations presented hereafter). The denominators for the horizontal forces (the moments) are the same. Therefore, the interactions between the two horizontal forces (moments) are described by circles.

**Loading surface.** The loading surface used was initially developed in the work of [Crémer, 2001] to describe the behaviour of a 2D shallow foundation. The adaptation for the 3D macro element is again simple, because of the circular shape of the footing. It consists in adding two terms in relation with the horizontal force \( H'_x \) and the moment \( M'_y \). One finally obtains the 5D surface given by Eq. (2.7). In Eq. (2.7) \( \bar{\varepsilon} = (\alpha, \beta, \delta, \eta) \) is the kinematic hardening vector composed of the four kinematics hardening variables and \( \rho \) the isotropic hardening variable. The variable \( \gamma \) is chosen to parameterize the second intersection point of the loading surface with the \( V' \) axis (the other point is the origin of the space) and its evolution in the \( V' \) axis. This hardening variable gives the maximum vertical load that the structure supported throughout the whole history of the loading (most of the time it is equal to the weight of the structure).

$$f_\varepsilon(\bar{\varepsilon}, \bar{\rho}) = \left( \frac{H'_x}{\rho \delta V'' (\gamma - V')} - \frac{\alpha}{\rho} \right)^2 + \left( \frac{M'_y}{\rho \delta V'' (\gamma - V')} - \frac{\beta}{\rho} \right)^2$$

$$+ \left( \frac{H'_y}{\rho \delta V'' (\gamma - V')} - \frac{\delta}{\rho} \right)^2 + \left( \frac{M'_x}{\rho \delta V'' (\gamma - V')} - \frac{\eta}{\rho} \right)^2 - 1 = 0$$

**Kinematic hardening rules.** Kinematic variables \( \alpha, \beta, \delta, \eta \) permit to determine the centre of the ellipse in the hyper plane \((H'_x, M'_y, H'_y, M'_x)\). The evolution of these variables has been obtained by studying the experimental and numerical behaviour of a
foundation under a monotonic static loading. More specifically, [Gottardi et al., 1999] provide the relations for a circular footing and for different kinds of soils (obtained from experimental tests) and [Crémer, 2001] uses similar curves (obtained with FEM simulations) to fit her model.

Assuming the classical partition of the total displacement $\hat{u}$ into an elastic part $\hat{u}^{el}$ and a plastic part and considering that $\hat{F} = K^{el} \hat{u}^{el}$, it is easy to link the increment of the forces with the increment of the associated plastic displacements. Then, as the behaviour is different for $\hat{F} > 0$ and $\hat{F} < 0$, two families of kinematic hardening laws and variables are used to describe the evolution of each force. 8 relations and variables are therefore used in the model for the eight forces $\hat{x}_H > 0, \hat{x}_H < 0, \hat{y}_H > 0, \hat{y}_H < 0, \hat{M}_x > 0, \hat{M}_x < 0, \hat{M}_y > 0, \hat{M}_y < 0$. For example for a radial loading, each kinematic hardening variable has the following expression (only the case of $\beta$ is presented below for simplicity):

$$\theta_{\beta} = \begin{cases} \frac{1}{b^{*} \beta_0 \gamma} K^{pl} \left( \frac{M_{\beta}^{*}}{\beta_0} - 1 \right) \theta_{\beta}^{pl} \bigg| \theta_{\beta}^{pl} = \infty, \\
-\frac{1}{b^{*} \beta_0 \gamma} K^{pl} \left( -\frac{M_{\beta}^{*}}{\beta_0} - 1 \right) \theta_{\beta}^{pl} \bigg| \theta_{\beta}^{pl} = \infty. \end{cases} \tag{2.8}$$

Where $M_{\beta}^{*}$ is the limit of the curve $\beta(\theta_{\beta}^{pl})$ when $\theta_{\beta}^{pl}$ tends to infinity. The evolutions of the other kinematic hardening variables are driven by similar relations.

The tangency rule defined in [Grange et al., 2006a] and [Grange et al., 2007a] provides the value of $M_{\beta}^{*}$.

**Isotropic hardening rule.** The independence of the directions (for $\hat{M} > 0$ and $\hat{M} < 0$) is taken into account using the specific kinematic hardening laws described in the previous paragraphs. However, one can also link the isotropic with the kinematical hardening laws [Crémer, 2001]. Indeed, when a plastic state is reached during a new cycle, the plastic behaviour is recovered at the same state (and with the same slope) as before. The evolution of the loading surfaces describing this property is given in Figure 2.14.

This property is translated into the following mathematical relation:
(c) Evolution of $\gamma$. Its evolution is closely dependant on the evolution of the vertical force $V'$. During the initialization phase, where the foundation is submitted only to the weight of the structure, $\gamma = V'$. After this first phase, the evolution of $\gamma$ is driven by the empirical relationship linking the vertical force and the vertical displacement given by [Nova et al., 1991]. Nevertheless, the other plastic displacements (horizontal displacements and rotations) can also increase the size of the loading surfaces in the direction of $V'$. Consequently, the evolution of $\gamma$ depends also on them according to the following expression:

$$\dot{\gamma} = K^{vl} \left( a_1 \dot{\gamma}^{pl} + a_2 \left| \dot{\gamma}^{pl} \right| + a_3 \left| \theta^{pl} \right| + a_4 \left| \dot{\theta}^{pl} \right| + a_5 \left| \delta^{pl} \right| \right) \left( 1 - \gamma \right)$$

(2.10)

Where $a_1, a_2, a_3, a_4$ and $a_5$ are parameters which permit to adjust the influence of each component of the plastic displacement vector.

(f) Flow rule. The normality rule is defined as: $\dot{\hat{u}}^{pl} = \left\{ \dot{\lambda} \right\} \frac{\partial g}{\partial F}$ where $g$ is the flow rule and the plastic multiplier $\left\{ \lambda \right\} = \dot{\lambda}$ if $\dot{\lambda} \geq 0$ and $\left\{ \lambda \right\} = 0$ if $\dot{\lambda} < 0$. A non associative flow rule is necessary [Crémer, 2001]. The flow rule $g$ used is defined by the following expression:
Figure 2.15: Flow rule

The representation of $g$ in a plane $(M', V')$ is given by the Figure 2.15. A similar figure is valid in planes $(H', V')$. The horizontal tangent of the flow rule can be adjusted using the 2 parameters $\xi$ and $\kappa$ in order to modify the evolutions of the plastic displacements in the hyper plane $(x^u, y^u, \theta^u, x^\theta, y^\theta, \theta^\theta)$.

2.1.3.2 Numerical simulations

The 3D macro element is implemented into FedelasLab, a finite element Matlab toolbox [Filippou et al., 2004]. The return mapping algorithm [Simo et al., 1998] is used for the plasticity mechanism. Two different simulations are provided hereafter:

- The aim of the first simulation is to see whether the macro element is able to give good results under a static loading. Numerical simulations are compared to experimental data coming from the works of [Gottardi et al., 1999].
- The second simulation uses the experimental results of the NEES 7 story building submitted to a dynamic loading [NEES7story, 2006].

Monotonic static behaviour

Detailed presentation of the tests is presented in [Gottardi et al., 1999]. They concern a circular footing of diameter $2R=D=0.1$m lying on a sand of a known density.

At the beginning, a vertical displacement is applied at the foundation until a given vertical force is reached. Then, the vertical displacement is kept constant while another displacement (horizontal displacement or rotation or a combined displacement) starts increasing. The tests are thus completely displacement controlled. The response of the foundation is represented in the space of forces. The curves described in the space...
\((H_x', M_y', H_x', M_y')\) are an approximation of the yield surface (that’s the reason why the tests are called “swipe tests”).

More details about the numerical simulations are presented in [Grange et al., 2006a], [Grange et al., 2007a], and [Grange et al., 2007b]. Two tests in [Gottardi et al., 1999] are simulated hereafter: the GG03 and GG07. During the GG03 test, an initial vertical force \(V=1600N\) is applied followed by an increasing horizontal displacement. During the GG07 test, once a vertical force \(V=1600N\) is reached it is reduced to \(V=200N\). After that, an increasing horizontal displacement is applied.

Figure 2.16(a, b) shows that the 3D macro element reproduces correctly both tests. It is interesting to note that the load path follows particularly well the failure criterion.

In order to show the behaviour of the macro element under a 3D loading, the following numerical 3D swipe test is performed. Figure 2.16(c, d, e) shows the load path in the \(\langle M_x/2R M_y/2R V \rangle\) space. At the beginning, a vertical displacement is imposed till a constant value. After that, the foundation is driven with an increasing rotation \(\theta_y\) until the moment \(M_y'\) reaches a given value. Finally, \(\theta_y\) is kept constant and a new increasing rotation \(\theta_x\) is applied to the foundation. Moments in the 2 directions are clearly developed and at the end the load path is very close to the failure surface.

Dynamic Behaviour – NEES 7 storey Building
In order to evaluate the efficiency of the macro element to predict the behaviour of a slender structure submitted to a dynamic loading, the simulation of the NEES 7 storey building experiment [NEES7story, 2006] is carried out. At first, the structure is considered embedded on the shaking table. A parametrical study is then presented on the influence of SSI (using six different types of soils).

Test description

The aim is to test a full-scale vertical slice of a seven-story reinforced concrete walls building (Figure 2.17) subjected to increasing intensity of uniaxial earthquake ground motions (from EQ1 to EQ4) on the new NEES Large High-Performance Outdoor shake table. The structure is composed of 2 main perpendicular walls: the web wall and the flange wall linked with the slabs. A pre-cast column needed to limit torsional behaviour and gravity columns to support slabs are also present. Only the direction Y of loading is considered (parallel to the web wall).

**Finite element mesh**

A simple finite element mesh is used for the spatial discretization of the structure. Masses are concentrated at each floor and the building is modelled using Timoshenko multifibre beam elements ([Kotronis et al., 2005] and [Mazars et al., 2006]). Constitutive material laws are based on damage mechanics to describe cracking of concrete [La Borderie, 1991] and on plasticity for steel ([Menegotto et al., 1973] modified by Filippou (UC Berkeley) with an isotropic hardening. Details about masses and the elastic properties of the beam elements are presented in [Grange et al., 2006b] and [Grange et al., 2007c].
Simulations considering the structure fixed to the shaking table

For the calculations presented in Figure 2.18 the structure in considered fixed at the shaking table (the 3D macro element is not used and the shaking table is simulated using horizontal elastic beams). Comparison between the numerical and the experimental lateral displacements at the top of the structure for EQ1 and EQ4 sequences show that the curves are in phase and that the peaks are correctly reproduced.

Simulations considering different types of soils

The 3D macro element is now introduced at the bottom of the numerical model (the horizontal beams used for the shaking table are omitted). The macro element represents a circular foundation of diameter equal to $2R=4m$. This foundation corresponds appreciatively to 2 rectangular footings with the dimensions given in Figure 2.19.
Different types of soils are considered in the following simulations. Their characteristics are defined in Table 2.3. All soils have a density $\rho = 1900 \text{kg/m}^3$ and a Poisson coefficient $\nu = 0.4$. Their properties are going from very low characteristic soil to a soil with very good characteristics according to the Eurocode 8 classification [CEN, 2004].

Table 2.3: Characteristics of the soils used for the SSI simulations.

<table>
<thead>
<tr>
<th>Soil reference</th>
<th>Shear Modulus $G_0$ shear wave velocity $V_s$</th>
<th>cohesion $c'$, friction angle $\phi'$</th>
<th>Stiffness of the system “soil/ circular foundation” [Gazetas, 1991]</th>
<th>ultimate compressive stress $q_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil 1: very low characteristics (class S2)</td>
<td>$V_s = 30 \text{m/s}$, $G_0 = 1.71 \text{MPa}$</td>
<td>$c' = 30 \text{kPa}$, $\phi' = 0$</td>
<td>$K_{\theta\theta} = 60.8 \text{MN/m}$, $K_{zz} = 22.8 \text{MN/m}$, $K_{hh} = 17.1 \text{MN/m}$</td>
<td>$q_{\text{max}} = 0.2 \text{MPa}$</td>
</tr>
<tr>
<td>Soil 2: low characteristics (class S1)</td>
<td>$V_s = 100 \text{m/s}$, $G_0 = 19 \text{MPa}$</td>
<td>$c' = 35 \text{kPa}$, $\phi' = 0$</td>
<td>$K_{\theta\theta} = 675.5 \text{MN/m}$, $K_{zz} = 253.3 \text{MN/m}$, $K_{hh} = 190 \text{MN/m}$</td>
<td>$q_{\text{max}} = 0.234 \text{MPa}$</td>
</tr>
<tr>
<td>Soil 3: average characteristics (class D)</td>
<td>$V_s = 175 \text{m/s}$, $G_0 = 58.18 \text{MPa}$</td>
<td>$c' = 15 \text{kPa}$, $\phi' = 30^\circ$</td>
<td>$K_{\theta\theta} = 2068 \text{MN/m}$, $K_{zz} = 775.7 \text{MN/m}$, $K_{hh} = 581.8 \text{MN/m}$</td>
<td>$q_{\text{max}} = 1.05 \text{MPa}$</td>
</tr>
<tr>
<td>Soil 4: good characteristics (class C)</td>
<td>$V_s = 300 \text{m/s}$, $G_0 = 171 \text{MPa}$</td>
<td>$c' = 30 \text{kPa}$, $\phi' = 30^\circ$</td>
<td>$K_{\theta\theta} = 6080 \text{MN/m}$, $K_{zz} = 2280 \text{MN/m}$, $K_{hh} = 1710 \text{MN/m}$</td>
<td>$q_{\text{max}} = 1.7 \text{MPa}$</td>
</tr>
<tr>
<td>Soil 5: very good characteristics (class B)</td>
<td>$V_s = 400 \text{m/s}$, $G_0 = 304 \text{MPa}$</td>
<td>$c' = 30 \text{kPa}$, $\phi' = 35^\circ$</td>
<td>$K_{\theta\theta} = 10808 \text{MN/m}$, $K_{zz} = 4053 \text{MN/m}$, $K_{hh} = 3040 \text{MN/m}$</td>
<td>$q_{\text{max}} = 2.88 \text{MPa}$</td>
</tr>
<tr>
<td>Soil 6: rock (class A): elastic stiffness of the shaking table</td>
<td>$V_s &gt; 800 \text{m/s}$</td>
<td></td>
<td>$K_{\theta\theta} = 18302 \text{MN/m}$, $K_{zz} = \infty$, $K_{hh} = \infty$</td>
<td>$q_{\text{max}} = \infty$</td>
</tr>
</tbody>
</table>

The numerical results considering the 6 different soils are presented in Figure 2.20. It shows the maximum overturning moments, story shears, lateral displacements, inter-storey drift ratios and floor accelerations for each level of the structure. The SSI influence is compared with the behaviour of the original structure embedded in the shaking table. The internal forces presented here (overturning moments and story shears) are calculated at the base of the web wall only.

As expected, the primary result of the SSI is that it isolates the structure. Indeed, when looking at Figure 2.20 one can see that overturning moments and story shears are reduced. For soils with low characteristics this reduction is more significant. Especially for soils 1 and 2 numerical predictions provide the location of the maximum moment...
near the level 2 and not at the base of the structure. Concerning the displacements at the top, they are higher for the structure on soils with low characteristics (1 and 2) than for the embedded structure. Nevertheless, they are lower for soils 3, 4 and 5. This seems to be the result of the influence of the 2nd vibration mode that becomes more significant for those three last cases [Grange et al., 2007c].

The inter story drift ratio for soils 1 and 2 is small, indicating that the structure remains elastic. This is also confirmed by the numerical results [Grange et al., 2007c].

![Graphs showing maximum overturning moments, story shears, lateral displacements, and inter-story drifts for the 6 different soils compared with the structure embedded on the shaking table](image)

### 2.1.3.3 Conclusions and suggestions for further developments

The 3D macro element developed within this work reproduces correctly the non linear behaviour of a circular rigid foundation lying on an infinite space submitted to a monotonic static, cyclic or dynamic loading. Using global variables it presents the advantage of inducing low computation costs. This first version of the 3D macro element is implemented in the Matlab toolbox FedeasLab.
Various improvements are possible, particularly concerning the fitting of the parameters (stiffness, shape of the loading surface) and the different rules (flow rule, tangency rule). The difficulty to develop a 3D macro element lies in the fact that it has to be capable of simulating non-radial loadings. Indeed, in 3D forces and moments in the two horizontal directions $x$ and $y$ are coupled via a nonlinear relation. The tangency rule that governs the evolution of the load surface, is thus complicated and needs to be improved.

2.1.4 Adaptive pushover analysis for 2D irregular buildings in 3D

2.1.4.1 Introduction

A major challenge in performance-based engineering is to develop simple, yet accurate methods for estimating seismic demand on structures considering their inelastic behaviour: the use of nonlinear static procedures, or pushover analyses, is inevitably going to be favoured over complex, impractical for widespread professional use, nonlinear time-history methods. The term 'pushover analysis' describes a modern variation of the classical 'collapse analysis' method. The procedure consists of an incremental-iterative solution of the static equilibrium equations corresponding to a nonlinear structural model subjected to a monotonically increasing lateral load pattern. The structural resistance is evaluated and the stiffness matrix is updated at each increment of the forcing function, up to convergence. The solution proceeds until (i) a predefined performance limit state is reached, (ii) structural collapse is incipient or (iii) the program fails to converge.

Within the framework of earthquake engineering, pushover analysis is employed with the objective of deriving, with relative ease, an envelope of the response parameters that would otherwise be obtained through a much more complex and time-consuming Incremental Dynamic Analysis (IDA) procedure, as can be construed by Figure 2.21. IDA is a parametric analysis method by which a structure is subjected to a series of nonlinear time-history analyses of increasing intensity [Vamvatsikos and Cornell, 2002], with the objective of attaining an accurate indication of the “true” dynamic response of a structure subjected to earthquake action.

According to recently introduced code provisions, pushover analysis should consist of subjecting the structure to an increasing vector of horizontal forces with invariant pattern. Both the force distribution and target displacement are based on the assumptions that the response is controlled by the fundamental mode and the mode shape remains unchanged until collapse occurs. Two lateral load patterns, namely the first mode proportional and the uniform, are recommended to approximately bound the likely distribution of the inertia forces in the elastic and inelastic range, respectively. However, a number of recent studies, partially summarised in the FEMA-440 report [ATC, 2005],
raise doubts on the effectiveness of these conventional force-based pushover methods in estimating the seismic demand throughout the full deformation range: (i) inaccurate prediction of deformations when higher modes are important and/or the structure is highly pushed into its nonlinear post-yield range, (ii) inaccurate prediction of local damage concentrations, (iii) inability of reproducing peculiar dynamic effects, neglecting sources of energy dissipation, viscous damping, and duration effects, (iv) difficulty in incorporating three-dimensional and cyclic earthquake loading effects.

In Figure 2.22, examples of inadequate prediction of the deformation response characteristics of a 12-storey reinforced concrete frame subjected to a natural earthquake recording (case-study RM15-NR2 in [Antoniou and Pinho, 2004a]) and of a 4-storey irregular frame subjected to an artificial accelerogram (ICONS full-scale test specimen, described in [Pinho and Elnashai, 2000]) are given. It is noted that although the 12-storey building is regular in height, its response is heavily influenced by higher mode effects, effectively rendering its seismic behaviour highly irregular in height, as conspicuously shown by Figure 2.22. The standard pushover results have been carried out using both triangular and uniform loading distributions, and are compared with the envelope of results obtained with incremental dynamic analysis.
2.1.4.2 Displacement-based Adaptive Pushover (DAP)

With a view to overcome the limitations described above, Antoniou and Pinho [2004b] have proposed a paradigm shift in pushover analysis, by introducing the innovative concept of Displacement-based Adaptive Pushover (DAP). Contrarily to what happens in non-adaptive pushover, where the application of a constant displacement profile would force a predetermined and possibly inappropriate response mode, thus concealing important structural characteristics and concentrated inelastic mechanisms at a given location, within an adaptive framework, a displacement-based pushover is entirely feasible, since the loading vector is updated at each step of the analysis according to the current dynamic characteristics of the structure.

It is worth recalling, or re-iterating, that in adaptive pushover the response of the structure is computed in incremental fashion, through piecewise linearisation. Therefore, it is possible to use the tangent stiffness at the start of each increment, together with the mass of the system, to compute modal response characteristics of each incremental pseudo-system through elastic eigenvalue analysis, and use such modal quantities to congruently update (i.e. increment) the pushover loading vector.

The implementation of DAP can be structured in four main stages; (i) definition of the nominal load vector and inertia mass, (ii) computation of the load factor, (iii) calculation of the normalised scaling vector and (iv) updating of the loading displacement vector. Whilst the first step is carried out only once, at the start of the analysis, its three remaining counterparts are repeated at every equilibrium stage of the nonlinear static
analysis procedure, as described in the following subsections.

The loading vector shape is automatically defined and updated by the solution algorithm at each analysis step, for which reason the nominal vector of displacements, \( U_0 \), must always feature a uniform (rectangular) distribution shape, in height, so as not to distort the load vector configuration determined in correspondence to the dynamic response characteristics of the structure at each analysis step. In addition, it is noteworthy that the adaptive pushover requires the inertia mass \( M \) of the structure to be modelled, so that the eigenvalue analysis, employed in updating the load vector shape, may be carried out.

The magnitude of the load vector \( U \) at any given analysis step is given by the product of its nominal counterpart \( U_0 \), defined above, and the load factor \( \lambda \) at that. The latter is automatically increased, by means of a load control strategy [Antoniou and Pinho, 2004a], until a predefined analysis target, or numerical failure, is reached. The normalized modal scaling vector, \( \bar{D} \), used to determine the shape of the load vector (or load increment vector) at each step, is computed at the start of each load increment. In order for such scaling vector to reflect the actual stiffness state of the structure, as obtained at the end of the previous load increment, an eigenvalue analysis is carried out. To this end, the Lanczos algorithm [Hughes, 1987] is employed to determine the modal shape and participation factors of any given predefined number of modes. Modal loads can be combined by using either the Square Root of the Sum of the Squares (SRSS) or the Complete Quadratic Combination (CQC) methods.

Since application to the analysis of buildings is the scope of the present work, use is made of the interstorey drift-based scaling algorithm, whereby maximum interstorey drift values obtained directly from modal analysis, rather than from the difference between not-necessarily simultaneous maximum floor displacement values, are used to compute the scaling displacement vector. This comes as a reflection of the fact that the maximum displacement of a particular floor level, being essentially the relative displacement between that floor and the ground, provides insufficient insight into the actual level of damage incurred by buildings subject to earthquake loading. On the contrary, interstorey drifts, obtained as the difference between floor displacements at two consecutive levels, feature a much clearer and direct relationship to horizontal deformation demand on buildings. Readers are referred to Antoniou [2002] for further details on this formulation. In such an interstorey drift-based scaling technique, the eigenvalue vectors are thus employed to determine the interstorey drifts for each mode \( \Delta_b \), while the displacement pattern \( D_i \) at the ith storey is obtained through the summation of the modal-combined inter-storey drifts of the storeys below that level, i.e. drifts \( \Delta_1 \) to \( \Delta_i \):
\[
\Delta U_t = \Delta \lambda_t \times \text{normalised shape at step } t \times \text{nominal load vector} = \text{new increment of displacements}
\]

\[
U_t = U_{t-1} + \Delta U_t = \text{balanced displacements} + \text{new increment of displacements} = \text{new displacements applied at step } t
\]

Figure 2.23 Updating of the loading displacement vector

Since only the relative values of storey displacements \((D_i)\) are of interest in the determination of the normalised modal scaling vector \(\overline{D}\), which defines the shape, not the magnitude, of the load or load increment vector, the displacements obtained above are normalised so that the maximum displacement remains proportional to the load factor, as required within a load control framework. Once the normalised scaling vector and load factor have been determined, and knowing also the value of the initial nominal load vector, the loading vector \(U_t\) at a given analysis step \(t\) is obtained by adding to the load vector of the previous step, \(U_{t-1}\) (existing balanced loads), a newly derived load vector increment, computed as the product between the current load factor increment \(\Delta \lambda_t\), the current modal scaling vector \(\overline{D}_t\) and the nominal vector \(U_0\), as graphically depicted in Figure 2.23. The DAP algorithm was implemented in the FE analysis program SeismoStruct [SeismoSoft, 2005], freely available from the internet.

Figure 2.24 Interstorey drift profiles of (a) 12-storey building and (b) 4-storey irregular frame, obtained with Displacement-based Adaptive Pushover using SRSS combination
2.1.4.3 Case-study results

In Figure 2.24, the interstorey drift profiles of the two case-studies being considered in this work, as obtained with the employment DAP analyses, are given. It is observed that the predictions now match much closer the dynamic response of these two structures, which effectively means that the response irregularities caused by the flexibility of the 12-storey structure, and subsequent amplification of higher modes, as well as the strength irregularity of the 4-storey prototype, have been fully and correctly captured by the proposed static analysis algorithm. In Figure 2.25, on the other hand, the capacity curves of the 12-storey building, as derived by both DAP and standard pushover curves are compared with the Incremental Dynamic Analysis envelope. The advantages of using an adaptive displacement-based pushover can be inferred also from this type of results.

![Figure 2.25 Capacity curves of a 12-storey building, obtained with DAP and standard pushovers, and compared against IDA envelopes](image)

The reasons behind this significantly improved performance lay on the fact that in displacement-based adaptive pushover storey forces or shears are no longer applied directly to the structure but rather come as a result of structural equilibrium to the applied displacement pattern, thus allowing for the reproduction of reversal of storey shear distributions, observed in dynamic analysis, even if a quadratic rule is employed to combine the contribution of the different modes. In effect, DAP drift profiles, despite carrying a permanently positive sign, do, in any case, feature changes of their respective gradient (i.e. the trend with which drift values change from one storey to the next), introduced by the contribution of higher modes. When such gradient variations imply a
reduction of the drift of a given storey with respect to its adjacent floor levels, then the corresponding applied storey horizontal force must also be reduced, in some cases to the extent of sign inversion [e.g. Pinho and Antoniou, 2005].

The effectiveness of DAP has also been verified on a series of extensive parametric studies on reinforced concrete frames [Pinho et al., 2005], steel buildings [Pinho et al., 2007], and reinforced concrete bridges [Pinho and Casarotti, 2007].

2.1.4.4 Concluding remarks

When compared with nonlinear time-history analysis, pushover methods are advantageous by their (i) higher user-friendliness, (ii) reduced running time and (iii) increased numerical stability. Therefore, it is important that the proposed displacement-based algorithm, capable of producing improved structural response predictions in comparison with existing non-adaptive pushover techniques, does also feature these three advantages over dynamic analysis.

From a usability point-of-view, the proposed displacement-based adaptive pushover algorithm effectively presents no additional effort and/or requirements with respect to its conventional non-adaptive counterparts. In effect, the only element of novelty, in terms of analysis input, is the introduction of the building’s inertia mass, which, however, can readily be obtained directly from the vertical gravity loads, already included in any type of pushover analysis.

With regards to computational effort, in general, the amount of time necessary to complete an adaptive pushover analysis is typically double the time necessary for a conventional procedure, approximately. Obviously, the duration of such finite element runs will vary according to the computing capacity of the workstation being used, as well as with the characteristics of the model (mainly the number of elements and level of fibre discretisation of the sections). In any case, adaptive pushover proved to be up to ten times quicker than nonlinear dynamic analysis of a same model (keeping in mind that fibre-based finite element modelling has been adopted for the current work), hence the time-advantage of static methods versus their dynamic counterparts is not lost with the addition of the adaptive features.

Regarding numerical stability, no particular problems are to be reported, noting that structures were pushed well into their post-peak inelastic response range (3% total drift).
2.2 TOOLS FOR ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BUILDINGS

2.2.1 Effective elastic stiffness of RC members for use in linear analyses emulating nonlinear ones

2.2.1.1 Introduction

Recent standards or guidelines for the seismic assessment and retrofitting of existing buildings are fully displacement-based, using, be it indirectly, member deformations for the assessment and retrofit-design of members. They also determine the seismic input in terms of global displacement demands, on the basis of the equal displacements rule. Their implementation requires procedures for simple, yet accurate, estimation of inelastic displacement and deformation demands throughout the structure; as a matter of fact, these new standards or guidelines accept estimation of the inelastic local displacement and deformation demands throughout the structure through linear analysis. The main condition for this very convenient approximation is to use a realistic estimate of the global elastic stiffness. For concrete structures this means using realistic values of the effective cracked stiffness of concrete members at yielding: if the analysis uses elastic member stiffness that reproduces well the secant member stiffness to yielding, a linear-elastic analysis with 5% damping can approximate well even the distribution of inelastic seismic displacement and deformation demands throughout the structure.

Within the force- and strength-based design philosophy of current seismic design codes, it is more conservative to use a high estimate of the effective stiffness, as this reduces the period and hence increases the corresponding spectral acceleration for which the structure is designed. Eurocode 8 and current US codes allow taking the stiffness of concrete members equal to 50% of that of the uncracked member, $E_c I_c$, neglecting the reinforcement. The default value of $0.5(E_c I_c)$ is conservative for force- and strength-based seismic design, but very unconservative in displacement-based seismic assessment and retrofitting of existing structures, or in displacement-based seismic design of new ones.

The secant stiffness to yielding of concrete members is commonly determined assuming purely flexural behaviour, from section moment-curvature relations and integration along the member. Such a calculation does not account for the effects of shear and inclined cracking, of anchorage slip, etc. The effective stiffness to yielding of concrete members depends on the longitudinal reinforcement; in displacement-based seismic design of new structures, it can not be reliably estimated before the seismic response analysis, as at that stage member reinforcement has not been determined yet.

In Section 2.2.1 two values of the “effective stiffness” to yielding of concrete members
are established for predictive purposes:

- a “theoretical effective stiffness”, calculated from the values of the yield moment, \( M_y \), and of the chord rotation, \( \theta_y \), at the yielding end obtained from first principles; and
- an “empirical effective stiffness”, fitted directly to test results on the “experimental effective stiffness”; the ratio of the “empirical effective stiffness” to the uncracked member stiffness, \( E_c I_c \), is given in terms of geometric and other characteristics of the member, known before dimensioning and detailing of its reinforcement.

The purely “empirical effective stiffness” predicts the experimental effective stiffness with just a little more scatter and lack-of-fit compared to the “theoretical” stiffness.

### 2.2.1.2 Definition and determination of effective stiffness

The effective elastic stiffness of the shear span of a RC member (moment-to-shear ratio at the end, \( L_s = M/V \)) in a bilinear force-deformation model under monotonic loading, may be taken as the value of the secant stiffness of the shear span at member yielding:

\[
E I_{\text{eff}} = \frac{M_y L_s}{30 \theta_y}
\]  

(2.11)

where \( M_y \) is the yield moment in the bilinear \( M-\theta \) model of the shear span and \( \theta_y \) is the chord rotation at the yielding end (deflection of the other end of the shear span with respect to the tangent to the member axis at the yielding end, divided by the shear span).

If experimental values of \( M_y \) and \( \theta_y \) are used in Eq.(2.11), the “experimental effective stiffness” at member yielding is obtained. Two values of the effective stiffness are established here for predictive purposes:

- A “theoretical effective stiffness”, calculated from Eq.(2.11) using the yield moment, \( M_y \), and the chord rotation, \( \theta_y \), at the yielding end obtained from first principles (with possible empirical corrections, after calibration with test results on \( M_y \) and \( \theta_y \)). As it requires knowledge of the amount and arrangement of reinforcement, the “theoretical effective stiffness” is appropriate for use in displacement-based seismic assessment of existing buildings.
- An “empirical effective stiffness”, fitted directly to test results on the “experimental effective stiffness”, in terms of geometric and other characteristics of the member known before dimensioning and detailing of its reinforcement. Although less accurate and robust than the “theoretical effective stiffness”, the “empirical” one is convenient for displacement-based seismic design of new structures.
Table 2.4: Mean*, median* and coef. of variation of ratio experimental-to-predicted values at yielding under uniaxial loading

<table>
<thead>
<tr>
<th>Quantity</th>
<th># of data</th>
<th>mean*</th>
<th>median*</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{y,exp}/M_{y,pred}$.1st-principles slender beam/columns</td>
<td>1844</td>
<td>1.045</td>
<td>1.025</td>
<td>16%</td>
</tr>
<tr>
<td>$M_{y,exp}/M_{y,pred}$.1st-principles slender rectangular walls</td>
<td>109</td>
<td>1.03</td>
<td>1.03</td>
<td>14.7%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.20a)}$. beam/columns - w/o slip</td>
<td>281</td>
<td>1.08</td>
<td>1.045</td>
<td>26.3%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.20a)}$. beam/columns - w/ slip</td>
<td>1276</td>
<td>1.06</td>
<td>1.015</td>
<td>33.1%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.20a)}$. beam/columns – all</td>
<td>1557</td>
<td>1.065</td>
<td>1.025</td>
<td>32%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.20a)}$. rectangular walls (all w/ slip)</td>
<td>98</td>
<td>0.995</td>
<td>0.995</td>
<td>37.9%</td>
</tr>
<tr>
<td>$(M_{y,L}/3\theta_{y})<em>{exp}/(M</em>{y,L}/3\theta_{y})_{pred}$. beam/columns</td>
<td>1520</td>
<td>1.04</td>
<td>0.995</td>
<td>32.2%</td>
</tr>
<tr>
<td>$(M_{y,L}/3\theta_{y})<em>{exp}/(E</em>{eff}(\theta,3X12)$ beam/columns</td>
<td>1520</td>
<td>1.045</td>
<td>1.005</td>
<td>36.8%</td>
</tr>
<tr>
<td>$(M_{y,L}/3\theta_{y})<em>{exp}/(M</em>{y,L}/3\theta_{y})_{pred}$. rectangular walls</td>
<td>98</td>
<td>1.205</td>
<td>0.97</td>
<td>57.0%</td>
</tr>
<tr>
<td>$(M_{y,L}/3\theta_{y})<em>{exp}/(E</em>{eff}(\theta,3X12)$. rectangular walls</td>
<td>98</td>
<td>1.04</td>
<td>1.00</td>
<td>47.0%</td>
</tr>
</tbody>
</table>

*If the sample size is large, the median is more representative of the average trend than the mean, as the median of the ratio predicted-to-experimental value is always the inverse of the median of the ratio experimental-to-predicted value, whereas the mean of both ratios is typically greater than the median.

2.2.1.3 

Theoretical effective stiffness

Calculation of yield moment from first principles

The yield moment, $M_{y}$, can be computed from equilibrium of the section, using the Navier-Bernoulli plane-section hypothesis, as:

$$
\frac{M_{y}}{bd} = \varphi_{y} \left\{ \frac{E_{c}}{2} \left[ 0.5(1+\delta_{1}) - \frac{\xi_{y}}{3} \right] + \frac{E_{t}}{2} \left[ (1-\xi_{y}) \rho_{1} + (\xi_{y} - \delta_{2}) \rho_{2} + \frac{\rho_{c}}{6} (1-\delta_{1})(1-\delta_{2}) \right] \right\} \left(2.12\right)
$$

where:

- $b$ is the width of the compression zone;
- $d$ is the section effective depth;
- $\varphi_{y}$ is the yield curvature;
- $\rho_{1}$ and $\rho_{2}$ are the ratios of the tension and the compression reinforcement (normalized to $bd$);
- $\rho_{c}$ is the ratio of “web” reinforcement, i.e. of the reinforcement (almost) uniformly

```
distributed between the tension and the compression steel (also normalized to $bd$);

$$\delta_1 = \frac{d_1}{d},$$

where $d_1$ is the distance of the centre of the compression reinforcement from the extreme compression fibres;

$E_s$ is the steel elastic modulus;

$E_c$ is the elastic modulus of concrete; and

$\xi_y$ is the compression zone depth at yielding (normalized to $d$).

If section yielding is identified by yielding of the tension steel, the yield curvature is:

$$\phi_y = \frac{f_y}{E_s(1 - \xi_y)d}$$

(2.13)

with $\xi_y$ given from:

$$\xi_y = \left(\alpha^2 A^2 + 2\alpha B\right)^{1/2} - \alpha A$$

(2.14)

in which $\alpha = E_s/E_c$ denotes the ratio of elastic moduli and $A, B$ are given by:

$$A = \rho_1 + \rho_2 + \rho_v + \frac{N}{bdf_y}, \quad B = \rho_1 + \rho_2 \delta_1 + 0.5 \rho_v (1 + \delta_1) + \frac{N}{bdf_y}$$

(2.15)

$N$ is the axial load (with compression considered as positive).

Apparent yielding of members with high axial load ratio, $\nu = N/Af_c$, is due to nonlinearity of the concrete in compression before yielding of the tension steel. Available test results on yielding of such members suggest it can be identified with exceedance of the following strain at the extreme compression fibres, while considering both steel and concrete as linear-elastic till then:

$$\varepsilon_c \approx \frac{1.8 f_c}{E_c}$$

(2.16)

Then apparent yielding of the member takes place at a curvature:

$$\varphi_y = \frac{\varepsilon_c}{\xi_y d} \approx \frac{1.8 f_c}{E_c \xi_y d}$$

(2.17)
where $\xi_y$ is still given by Eq.(2.14), but this time with $A, B$ given by:

$$A = \rho_1 + \rho_2 + \rho_v - \frac{N}{\epsilon_c E_b d} \approx \rho_1 + \rho_2 + \rho_v - \frac{N}{1.8 \alpha b d f_y}, \quad B = \rho_1 + \rho_2 \delta + 0.5 \rho_v (1 + \delta) \quad (2.18)$$

The lower of the two $\phi_y$ values from Eqs.(2.13) or (2.16) is the yield curvature.

As the corner of a bilinear $M-\theta$ curve enveloping the measured hysteresis loops expresses global yielding of the member and is slightly past 1st yielding of the extreme tension steel or compression fibres, the “experimental yield moment” at the corner of a bilinear $M-\theta$ curve fitted to the envelope of the measured $M-\theta$ loops in 1844 tests on slender beams/columns and 109 tests on slender walls free of shear effects exceeds the prediction of Eqs. (2.12)-(2.18) by an average factor of 1.025 or 1.03 for beams/columns or walls, respectively (cf. Table 2.4). This factor should be applied as correction factor on the value of $\rho_s$ from Eqs.(2.13)-(2.18).

**Fixed-end rotation due to rebar pull-out from their anchorage beyond the section of maximum moment, at yielding at that section**

In tests the curvature is generally measured as relative rotation between the section of maximum moment and a nearby section, divided by the distance between the two sections. Often measured relative rotations include the “fixed-end rotation” of the section of maximum moment due to slippage (pull-out) of longitudinal bars from their anchorage beyond that section. Even when this is the case, rotations measured between the section of maximum moment and two different member sections (different gauge lengths) allow estimation of the “fixed-end rotation” due to slippage (pull-out) of longitudinal bars.

At yielding at the section of maximum moment, the tension steel is at its yield value, $f_y$, at that section. Fixed-end rotation due to bar pull-out is equal to the slip from the anchorage zone, divided by the depth of the tension zone at yielding, which at yielding of the tension steel is equal to $(1-\xi)d$. Assuming that bond stresses are uniform over a length $h$ of the bar beyond the section of maximum moment, when the tension reinforcing bars yield at that section their tensile stress decreases linearly along the length $h$, from $f_y$ at the section of maximum moment, to zero at the end of length $h$. Then bar slippage from its anchorage is equal to $0.5f_yh/E_s$. The ratio of $f_y/E_s$ to $(1-\xi)d$ gives $\phi_y$ (cf. Eq.(2.13)). The length $h$ is proportional to the bond stress demand at yielding of the tension steel, i.e. to the ratio of the bar yield force, $A_s f_y$, to its perimeter, $\pi d_b L$, i.e. to $0.25d_b f_y$ where $d_{bl}$ is the mean diameter of longitudinal bars) and inversely proportional to bond strength, i.e. to $\sqrt{f_c}$. Assuming a mean bond stress along the length $h$ equal to $\sqrt{f_c}$ (i.e., lower than the ultimate bond stress of about $2\sqrt{f_c}$ or $2.5\sqrt{f_c}$ in unconfined or
confined concrete, respectively), we obtain for the "fixed-end rotation" due to slippage of longitudinal bars at yielding at the section of maximum moment:

$$\theta_{y,\text{slip}} = \frac{\varphi_y d_{bl} f_y}{8\sqrt{f'_c}} \left( f'_t \text{ and } f'_c \text{ in MPa} \right).$$  \hspace{1cm} (2.19)

This relation gives good average agreement with experimentally measured or derived values (e.g., from measurements at different gauge length in the same test) of the “fixed-end rotation” at yielding at the section of maximum moment, due to slippage of rebars from their anchorage beyond that section.

**Chord rotation at yielding**

Theoretically, the part of $\theta$ which is due to flexural deformations alone is equal to $\varphi_y L_s/3$. Any inclined cracking and shear deformations along the shear span increase the magnitude of $\theta$. Diagonal cracking near the yielding end of the member spreads yielding of the tension reinforcement up to the point where the first diagonal crack from the end section intersects this reinforcement. In general, diagonal cracking shifts the value of the tension force in the tension reinforcement from the cross-section to which this value corresponds on the basis of the bending moment and axial force diagrams, to a section where the bending moment is lower (i.e. away from the member end). This is the basis of the “shift rule” in dimensioning of the tension reinforcement at the Ultimate Limit State in bending with axial force. Although the magnitude of the shift depends on the amount of transverse reinforcement and on the angle of the diagonal compression struts with respect to the member axis at ultimate shear failure by diagonal tension, normally a default value of the shift equal to the internal lever arm, $z$, is used. Such a shift increases the part of $\theta$ theoretically due to flexural deformations, from $\varphi_y L_s/3$ to approximately $\varphi_y (L_s + z)/3$ (strictly speaking to $\varphi_y [(L_s - z)(1 - z/L_s)(1 + 0.5z/L_s)/3 + z(1 - 0.5z/L_s)]$, but the difference with the simpler term $\varphi_y (L_s + z)/3$ is practically nil). Such an increase would not take place unless flexural yielding at the end section is preceded by diagonal cracking. So, the term $z$ is added to $L_s$ only when the shear force that causes diagonal cracking ($V_{zh}$) is less than the shear force at flexural yielding of the end section, $V_{zh} = M_i/L_s$.

The following expressions were derived from tests with yielding in flexure:

- for beams or rectangular columns:

$$\theta_y = \varphi_y \frac{L_s}{3} + a_t \frac{z}{3} + 0.0013 \left( 1 + 1.5 \frac{h}{L_s} \right) + a_{sl} \frac{\varphi_y d_{bl} f_y}{8\sqrt{f'_c}} \left( f'_t \text{ and } f'_c \text{ in MPa} \right).$$  \hspace{1cm} (2.20a)
for rectangular walls:

\[
\theta_y = \varphi_y \left( \frac{L_s}{3} + a_v \frac{a_v L_s}{8b} \right) + 0.002 \cdot \max \left( 0, \frac{L_s}{8b} \right) + a_d \frac{\varphi_y d h f_y}{8\sqrt{f_c}}
\]  

(2.20b)

In the 1st term of Eqs.(3.10) the value of \( \varphi_y \) is the “theoretical” yield curvature from Eqs.(2.13)-(2.18), times the correction factor of 1.025 or 1.03 for beams/columns or walls, respectively; \( a_v \) is a zero-one variable: \( a_v = 0 \) if \( V_{Rc} > V_{My} = M_y/L_s \) and \( a_v = 1 \), if \( V_{Rc} \leq V_{My} = M_y/L_s \); the length of the internal lever arm, \( z \), is taken equal to \( z = d - d_i \) in beams/columns and to \( z = 0.8h \) in rectangular walls. The value of \( V_{Rc} \) is taken according to EN-Eurocode 2 [CEN 2004b]: for \( b_w \) (width of the web) and \( d \) in m, \( f_c \) in MPa, \( \rho_1 \) denoting the tensile reinforcement ratio, and the axial load \( N \) (positive for compression, but if \( N \) is tensile, then \( V_{Rc} = 0 \)) in kN, then \( V_{Rc} \) (in kN) is given by:

\[
V_{Rc} = \max \left[ 180(100\rho_2)^{1/3}, 35 \sqrt{1 + \left( \frac{0.2}{d} f_y^{1/6} \right)^2 \left( 1 + \left( \frac{0.2}{d} f_y^{1/3} + 0.15 \frac{N}{A_y} \right) b_w d \right)} \right] \]  

(2.21)

- The 2nd term of Eqs.(2.20), attributed to shear deformations along the shear span, is purely empirical and results from the fitting to the data.

- In the 3rd term of Eqs.(2.20) (the fixed-end rotation from Eq.(2.19)), \( a_d \) is a zero-one variable, with \( a_d = 1 \) if slippage of longitudinal steel from its anchorage zone beyond the end section is possible, or \( a_d = 0 \) otherwise. Both \( f_y \) and \( f_c \) are in MPa.

Table 2.4 gives an overview of the fitting of the “experimental” chord rotation at yielding by Eqs. (2.20), as well as of Eq. (2.11) to the “experimental” effective stiffness” (which is the main target of this endeavour). Note that the statistics are practically the same, if the form of Eqs. (2.20) adopted in Part 3 of EN-Eurocode 8 [CEN 2005a] (namely one with coefficient 0.13 in the 3rd term, instead of 1/8=0.125) is used.

### 2.2.1.4 Empirical effective stiffness

Statistical correlation analysis shows that the ratio of the “experimental” effective stiffness, \( E_{Ist} \), to the uncracked gross section stiffness, \( E_{ist} \), depends mainly on the following member parameters that are known before dimensioning of its reinforcement:

1. The type of member (beam, column or wall);
2. The possibility of slippage of longitudinal bars from their anchorage beyond the member end section (this is always possible in real structures).
3. The shear span ratio, $L_s/h$, at the end of the member, which in practice may be estimated taking the shear span, $L_s$, equal to 50% of the member clear span between in beams or columns, or to 50% of the distance of the bottom section of the storey to the top of the wall, in walls.

4. the axial load ratio, $\nu = N/A_{fc}$, under gravity actions alone (the ones acting together with the seismic action of interest) which in practice can be estimated even without analysis, from the tributary floor area of the vertical element of interest.

Due to the limited sample size for most combinations of the discrete parameters no. 1 and 2 above (e.g., data without slippage of reinforcement are very few for columns and not available for walls, while data for beams are mostly for no slippage, etc.), the same dependence on parameters 2 to 4 is assumed for all types of members (parameter no. 1), up to a constant that takes different values depending on member type. The result is:

$$EI_{\text{eff}} = a \left(0.8 + \log_{10} \frac{L_s}{h}\right) \left[1 + 0.048 \min \left(\frac{N}{A_{fc}}, 50 \text{MPa}\right) \left(1 - 0.25a_{sl}\right)EI_{\text{gras}}\right]$$

(2.22)

where:

$a = 0.108$ for columns;

$a = 0.133$ for beams; and

$a = 0.152$ for rectangular walls.

$a_{sl} = 1$ if slippage of longitudinal steel from its anchorage zone beyond the end section is possible, or $a_{sl} = 0$ otherwise.

As shown by the statistics in Table 2.4, Eq.(2.22), although purely empirical and independent of the reinforcement, etc., gives the effective stiffness with just a little more scatter and lack-of-fit than the “theoretical” stiffness from Eq.(2.11).

### Effective stiffness under biaxial (bidirectional) loading

If yielding under biaxial loading is identified with the corner of a bilinear moment-chord rotation approximation of the envelope of measured hysteresis loops in each one of the two directions of bending, the few available bidirectional tests on RC columns show the following (see Table 2.5 for statistics of the ratio of experimental to predicted values):

1. The “experimental yield moments” in each one of the two directions of bending agree well on average with the components of flexural resistance under biaxial loading computed based on first principles (i.e. equilibrium, the plane-section
hypothesis and an elastic-perfectly plastic stress-strain law for the reinforcement and a parabolic one for the concrete up to $f_c$ and a strain of 0.002 and horizontal thereafter, up to a compressive strain of 0.0045 at one corner of the cross-section.

2. The “experimental” components of the chord rotation at yielding, $\theta_{yy}, \theta_{yz}$, exceed by about 10%, on average, a circular interaction diagram of the form of Eq. (2.23), with the chord rotations at yielding under unidirectional bending, $\theta_{yy,uni}, \theta_{yz,uni}$, estimated from Eqs. (2.19).

$$\left(\frac{\theta_{yy,exp}}{\theta_{yy,uni}}\right)^2 + \left(\frac{\theta_{yz,exp}}{\theta_{yz,uni}}\right)^2 = 1 \tag{2.23}$$

3. As a result of 2 above, the “experimental” effective elastic stiffness in each one of the two directions of biaxial loading is about 10% less, on average, than the uniaxial “theoretical effective stiffness” calculated from the theoretical moment and chord rotation at yielding in uniaxial loading. By contrast, the “experimental” effective elastic stiffness in each one of the two directions of biaxial loading exceeds by about 7% the “empirical effective stiffness”.

It is concluded that the limited biaxial test results do not show that the effective elastic stiffness in each one of the two directions of biaxial loading is systematically larger or smaller than the effective elastic stiffness in uniaxial loading, “theoretical” or “empirical”.

<table>
<thead>
<tr>
<th>Quantity at yielding under biaxial loading</th>
<th># of data</th>
<th>mean</th>
<th>median</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{yy,exp}/M_{yy,pred.-1st-principles}$</td>
<td>35</td>
<td>1.00</td>
<td>0.99</td>
<td>11.6%</td>
</tr>
<tr>
<td>$M_{yz,exp}/M_{yz,pred.-1st-principles}$</td>
<td>35</td>
<td>1.00</td>
<td>0.98</td>
<td>12.0%</td>
</tr>
<tr>
<td>SRSS of $\theta_{yy,exp}/\theta_{yy,Eq.(2.19)}$ &amp; $\theta_{yz,exp}/\theta_{yz,Eq.(2.19)}$</td>
<td>34</td>
<td>1.16</td>
<td>1.11</td>
<td>21.9%</td>
</tr>
<tr>
<td>$\langle M_{yy,1st principles} L_y/3 \theta_{yy,Eq.(2.19)} \rangle$</td>
<td>34</td>
<td>0.93</td>
<td>0.90</td>
<td>24.4%</td>
</tr>
<tr>
<td>$\langle M_{yz,1st principles} L_y/3 \theta_{yz,Eq.(2.19)} \rangle$</td>
<td>34</td>
<td>0.93</td>
<td>0.87</td>
<td>23.9%</td>
</tr>
<tr>
<td>$\langle M_{yy,L_y/3 \theta_{yy}} \rangle/\Pi_{eff,y}$ Eq.(2.22)</td>
<td>34</td>
<td>1.05</td>
<td>1.07</td>
<td>23.4%</td>
</tr>
<tr>
<td>$\langle M_{yz,L_y/3 \theta_{yz}} \rangle/\Pi_{eff,z}$ Eq.(2.22)</td>
<td>34</td>
<td>1.08</td>
<td>1.07</td>
<td>26.2%</td>
</tr>
</tbody>
</table>
2.2.2 Ductility-dependent equivalent damping for displacement-based design (DDBD)

2.2.2.1 Introduction

The concept of viscous damping is generally used to represent the energy dissipated by the structure in the elastic range. Such dissipation is due to various mechanisms such as cracking, nonlinearity in the elastic phase of response, interaction with non-structural elements, soil-structure interaction, etc. As it is very difficult and unpractical to estimate each mechanism individually, the elastic viscous damping represents the combined effect of all of the dissipation mechanisms. There is no direct relationship of such damping with the real physical phenomena. However, the adoption of the viscous damping concept facilitates the solution of the differential equations of motion, represented by Eq. (2.24):

\[ \ddot{u} + 2\xi \omega_n \dot{u} + \omega_n^2 u = 0 \]  

where

\[ \xi = \frac{c}{2m\omega_n} \]

\( u \) is the displacement, \( \omega_n \) is the natural vibration frequency (radians/sec) of the system and \( \xi \) is the damping ratio or fraction of critical damping.

As mentioned above, the assumption of viscous damping simplifies greatly the dynamic problem and this is the reason why in the direct displacement based design the non-linear behaviour has been also represented by including the hysteretic damping into the damping term of Eq.(2.24a). Using this approach it is possible to solve a simple linear system instead of a non-linear system which is more time and resource demanding for design applications.
2.2.2.2 **Existing equivalent viscous damping equations**

There are multiple references which report different equations for the equivalent viscous damping factor, including those by Priestley [2003], Fardis and Panagiotakos [1996], Miranda and Ruiz [2002], Calvi [1999]. Some are based on Jacobsen’s approach [1930, 1960], others on the substitute damping approach, and others on results of time-history analyses. Some of the reported equations are compared in Figure 2.26.

Miranda [2002] carried out an investigation comparing the capabilities of different performance based methodologies (including methodologies based on equivalent linearization) in estimating the inelastic displacement for Takeda, modified Clough, stiffness degrading and elastoplastic system using 264 ground motion records. Similarly to many other researchers [e.g. Gulkan and Sozen, 1974; Iwan, 1980; Judi et al., 2000; Kowalsky and Ayers, 2002; Priestley, 2003], the findings of Miranda also suggested that Jacobsen’s approach for estimating the equivalent viscous damping factor was non-conservative for structures with high hysteretic energy absorption.

This study therefore aims at carrying out additional research in order to complement previous studies and to optimise damping values applicable to a wide range of hysteretic models and period ranges. For the sake of brevity, however, only a summary of the results obtained will be included here, the reader being instead referred to other publications [e.g. Grant, 2005; Priestley and Grant, 2005; Priestley et al., 2007]

2.2.2.3 **Methodology for calibrating equivalent viscous damping for DDBD**

A methodology was implemented, to modify, where necessary, Jacobsen’s equations for
the equivalent viscous damping factor, $\xi$. The procedure adopted determined the value of equivalent damping that has to be applied to an equivalent elastic system with a given effective period (based on the secant stiffness to maximum displacement response) in order to match its response (in terms of maximum displacement) to that obtained from a system with the same period (effective period) and a given level of ductility using nonlinear time-history analysis. The final objective of this procedure was to develop equations that define the equivalent damping factor to be used in DDBD for a given level of ductility and hysteretic model. Six hysteretic models, depicted in Figure 2.27 were considered in the analysis:

- Modified Takeda Model [Loeding et al., 1998], employed in two different versions in the analyses. The first, a “thin” Takeda model with $\alpha = 0.5$ and $\beta = 0$ was considered appropriate for bridge piers and wall structures. The second “fat” Takeda model ($\alpha = 0.3, \beta = 0.6$) was intended to be appropriate for reinforced concrete frames. In both cases the post-yield stiffness ratio was taken as 0.05.

- Bilinear [Otani, 1981], intended to model a bridge structure isolated with friction pendulum dampers. The behaviour represented the composite flexibility of dampers plus bridge piers, and a high post-yield stiffness ratio of $r = 0.2$ was consequently adopted.

- Elastic-Perfectly Plastic [Otani, 1981], included solely because of its historic importance in seismic time-history analysis. Its closest approximation in real structures is a flexible structure isolated with a flat coulomb (friction) damper.

- Ramberg Osgood [Otani, 1981], selected as being reasonably appropriate for structural steel members.

- Ring-spring, chosen to represent a precast concrete structure connected with unbonded prestressing, resulting in a hysteretic model characterized by bilinear elastic response with low hysteretic damping.

Six different synthetic accelerograms were selected in order to carry out the time-history analysis of the SDOF systems. All of them were constructed matching a given displacement design response spectrum at 5% damping. The first accelerogram was a synthetic record based on a record from Manjil, Iran, 20th June 1990, developed by Bommer and Mendis [2005] for use specifically in this research. The rest of the accelerograms were artificial records obtained as part of this study and complemented with others obtained by Alvarez [2004] and Sullivan [2003]. Average spectra for 20 levels of damping were computed for values between 3% and 60%. The spectral value for a given damping was obtained by interpolation between the closest upper and lower spectra bounding it. All of the records were compatible with the ATC32 design spectrum for soil type C and PGA 0.7 g. [ATC, 1996]. The records were selected so that the results of the dynamic analyses are representative of a larger set of non-spectrum compatible records.
Equivalent viscous damping in DDBD is defined as the combination of two effects: the elastic damping and the hysteretic damping. The initial elastic viscous damping used for time-history analysis of SDOF systems has been traditionally defined in practice by use of a constant damping coefficient corresponding to 5% of critical damping, though lower values are sometimes used for steel structures. This value is assumed to represent the different sources of energy dissipation when the structure is considered in the elastic range. It is not clear that constant coefficient damping is appropriate for structures...
responding inelastically, since the hysteretic models generally incorporate the full structural energy dissipation in the inelastic range, and other contributory mechanism, such as foundation damping will be greatly reduced when the structure enters the inelastic range. It would appear that tangent-stiffness proportional damping would be more appropriate than constant coefficient (initial-stiffness proportional, or mass-proportional) damping in modelling initial elastic damping in seismic response. The adoption of different characteristic stiffness in DDBD (secant stiffness) and time-history analyses (initial stiffness) further confuses the issue.

The representation of elastic damping is often given little consideration, but as shown in Priestley and Grant [2005], the issue of how this damping should be represented is important, not just to DDBD, but also to time-history analysis. In this study the influence of elastic damping was removed from both the design process and the time-history validation, by specifying zero elastic damping.

Time-history analysis were carried out using the program Ruaumoko [Carr, 1996] using a Newmark constant average acceleration integration scheme with $\beta = 0.25$. As described above, this procedure was carried out iteratively until the displacement of the equivalent SDOF system was the same for the time-history and for the design spectral analysis. The time step used for the integration was taken as half of the discretization step of each accelerograms, this is, 0.005 seconds except for the Manjil adjusted record which has a discretization step of 0.0045.

Table 2.6 Constant values for equivalent viscous damping equation

<table>
<thead>
<tr>
<th>Constant</th>
<th>Takeda Thin</th>
<th>Takeda Fat</th>
<th>Bilinear</th>
<th>EPP</th>
<th>Ramberg Osgood</th>
<th>Ring Spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>95</td>
<td>130</td>
<td>160</td>
<td>140</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.45</td>
<td>0.5</td>
</tr>
<tr>
<td>c</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

2.2.2.4 **Design equations for equivalent viscous damping factor**

The equations proposed by Priestley [2003] were taken as the base for the modified equations proposed here. The resulting equations have the following form:

$$\xi_{\text{effective}} = \frac{a}{\pi} \left(1 - \frac{0.1 \cdot r \cdot \mu}{\mu} \right) \left(1 + \frac{1}{T_{\text{eff}} + \epsilon} \right) \cdot \frac{1}{N}$$  \hspace{1cm} (2.25)
where $a$, $b$, $c$ and $d$ are coefficients defined for each hysteretic model, $\mu$ is the ductility, $T_{eff}$ is the effective period, $r$ is the post–elastic stiffness coefficient (applicable only in the case of the bilinear model) and $N$ is a normalising factor. An important difference of this equation from previous existing proposals is the extra term which is dependent on effective period. An exception exists in the recommendations of Judi et al. [2002], which indicate weak period-dependency.

The process to obtain the calibration factors $a$, $b$, $c$, $d$ was carried out for each hysteretic model analyzed. Not all the correction factors depend in the same proportion on the variables (period and ductility). A perfect match was not possible for all the cases because it was necessary to keep a simple form of the equation. Table 2.6 shows the constants obtained for the hysteretic models analyzed.

It is important to mention that the implementation of this approach in DDBD will modify slightly the design process, since in the latter the damping is obtained directly from the ductility; then, the effective period is obtained for a given target displacement. However, using the modified equation it will be necessary to iterate in order to obtain the period. The additional work is, however, insignificant.

One alternative to this procedure could be derived from the fact that the dependency of the equivalent damping reduces very rapidly as the period increases. The damping becomes almost independent for a period longer than 1 sec. Using this approximation it would not be necessary to iterate in the design procedure as mentioned previously. However, given that the procedure is simple the proposed equation has the advantage of being general for any period. Reduced versions of the equation may be obtained easily according to the characteristics of the structures considered in the design.

2.2.2.5 Concluding remarks

The design displacements based on Jacobsen’s approach within the direct displacement-based design methodology were calculated for six different hysteretic models with different effective periods and displacement ductilities. These design displacements were compared with results from time-history analyses using six earthquake records compatible with the design spectra. Both designs and time-history analyses were carried out using zero elastic viscous damping to enable the contribution of hysteretic damping to be directly determined. As has been found in earlier studies, the results obtained were inconsistent, with the time-history displacements exceeding the design displacements in many cases, particularly for hysteretic models with high energy absorption.

An iterative procedure was used to determine the required value for equivalent viscous
damping to be used in direct displacement-based design to equate the design displacement and the time-history results. From these analyses a series of design equations determining the equivalent viscous damping as a function of hysteresis rule, displacement ductility and period were developed.
3 ESTIMATION OF COMPONENT FORCE AND DEFORMATION CAPACITIES IN BUILDINGS

3.1 ACCEPTANCE AND DESIGN CRITERIA IN TERMS OF DEFORMATIONS, FOR RC MEMBERS UNDER UNI- OR BI-DIRECTIONAL CYCLIC LOADING, AT DIFFERENT PERFORMANCE LEVELS

3.1.1 RC member deformation-based design criteria for Performance-based seismic design of buildings

“Performance-based earthquake engineering” aims at maximizing the utility from the use of an earthquake-resistant facility by minimizing its expected total cost, including that of construction and the expected cost of any consequences of failure (in terms of casualties, cost of repair, loss of use, etc.) under future seismic events. As taking into account all possible future seismic events, with their annual probability of occurrence, and convoluting with the corresponding consequences during the design working life of the facility is not feasible, at present “performance-based earthquake engineering” advocates just replacing the traditional single-level design against collapse and its prescriptive rules, by a transparent multi-level design to meet more than one discrete “performance levels”, each one under a different seismic event (identified with its annual probability of exceedance and termed “seismic hazard level”). The pairing of all “performance levels” considered in a particular case with the associated “seismic hazard levels” is termed in performance-based earthquake engineering “Performance objective”.

Following US documents for performance-based seismic evaluation of existing buildings [ASCE, 2000], three performance levels are considered in this Section:

- “Operational”
- “Life-safety”; and
- “Near collapse”.

For new buildings of ordinary importance, “Life Safety” performance is normally appropriate for a hazard level corresponding to the “design seismic action” of new structures, while the “Near Collapse” performance level should normally be satisfied for a very severe hazard level, about 1.5 times the “design seismic action” corresponding to the “Life Safety” performance level of such buildings.
Owing to the recent emergence of procedures for seismic assessment of existing structures that entail member verifications explicitly in terms of deformations [ASCE, 2000 and 2001, CEN, 2005a] and the codification of the design of new structures directly on the basis of nonlinear analysis with explicit checks of member deformations [CEN, 2004], there is a large interest of the international earthquake engineering community in the quantification of the deformation capacity of RC members. To address this problem, a databank of tests on RC members conducted all over the world has been built in the past ten years at the University of Patras. The present Section describes the outcome of the use of this database for the quantification of acceptable deformations at the different performance levels of interest in displacement- and performance-based seismic design of buildings, which are consistent with the uncertainty (scatter) of deformation capacity and take into account the target reliability level for the performance level of interest.

Table 3.1 shows the deformation-based design criteria proposed here for the verification of members, depending on the performance level at which design takes place.

Table 3.1: RC member verification and design criteria

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary ductile members</td>
<td>$\delta_C \leq \delta_{y}$</td>
<td>$\delta_C \leq \delta_{uk,0.05}$</td>
<td>$\delta_C \leq \delta_{um-a}$</td>
</tr>
<tr>
<td>Secondary ductile members</td>
<td>$\delta_C \leq \delta_{y}$</td>
<td>$\delta_C \leq \delta_{u,m-s}$</td>
<td>$\delta_C \leq \delta_{u,m}$</td>
</tr>
<tr>
<td>Primary brittle members</td>
<td>$V_{E,max} \leq V_{Rd}$</td>
<td>$V_{E,max} \leq V_{R,m-\sigma}$</td>
<td>$V_{E,max} \leq V_{R,m-\sigma}$</td>
</tr>
<tr>
<td>Secondary brittle members</td>
<td>$V_{E,max} \leq V_{Rd}$</td>
<td>$V_{E,max} \leq V_{R,m-\sigma}$</td>
<td>$V_{E,max} \leq V_{R,m-\sigma}$</td>
</tr>
</tbody>
</table>

where:
- $\delta_C$ is the deformation demand (chord rotation, $\theta$, or curvature, $\phi$) for the combination of the seismic action corresponding to the performance level of interest with the simultaneously acting gravity loads, from the analysis (inelastic or elastic, as appropriate);
- $V_{E,max}$ is the maximum shear force for the combination of the seismic action corresponding to the performance level of interest with the simultaneously acting gravity loads, from the analysis (inelastic or elastic, as appropriate);
- $V_{E,CD}$ is the maximum shear force estimated from capacity-design calculations, in case elastic analysis is used for the combination of the seismic action corresponding to the performance level of interest with the simultaneously acting gravity loads;
- $\delta_{y}$ is the expected value of the yield deformation (chord rotation, $\theta$, or curvature, $\phi$);
- $\delta_{uk,0.05}$ is the lower characteristic (5%-fractile) value of the deformation (chord rotation, $\theta$, or curvature, $\phi$) capacity;
- $\delta_{u,m}$ is the mean value of deformation (chord rotation, $\theta$, or curvature, $\phi$) capacity;
- $\delta_{u,m-\sigma}$ is the mean-minus-standard deviation value of the deformation (chord rotation,
\( \theta \), or curvature, \( \varphi \) capacity;
- \( V_{R_{\theta}} \) is the mean value of the shear force capacity, calculated on the basis of the nominal or characteristic values of material properties, \( f_{ck} \) and \( f_{yk} \);
- \( V_{R_{\theta,d}} \) is the design value of the mean shear force capacity, calculated from the design values of material properties, \( f_{cd} \) and \( f_{yd} \), equal to the nominal values, \( f_{ck} \) and \( f_{yk} \), divided by material (partial) factors \( \gamma_{m} \) (for enhanced reliability against brittle shear failure);
- \( V_{R_{\varphi}} \) is the mean-minus-standard deviation value of the shear force capacity, calculated from the nominal or characteristic values of material properties, \( f_{ck} \) and \( f_{yk} \);
- \( V_{R_{\varphi,d}} \) is the mean-minus-standard deviation value of the shear force capacity, calculated on the basis of design values of material properties, \( f_{cd} \) and \( f_{yd} \), equal to the nominal or characteristic values, \( f_{ck} \) and \( f_{yk} \), divided by material (partial) factors \( \gamma_{m} \).

In all the verifications, nominal values of material properties, \( f_{ck} \) and \( f_{yk} \), are used, except for some verifications of brittle mechanisms of force transfer and failure modes, notably of shear, in which the design value of the shear force capacity, \( V_{R_{\theta,d}} \), is used (calculated on the basis of design values of material properties, \( f_{cd} \) and \( f_{yd} \), equal to the nominal or characteristic values, \( f_{ck} \) and \( f_{yk} \), divided by material factors \( \gamma_{m} \)).

According to Table 3.1, members are checked:
- in flexure-controlled, ductile mechanisms of behaviour and failure, on the basis of the deformations (chord rotations, \( \theta \), or curvatures, \( \varphi \)) in flexural plastic hinges,
- in shear-controlled, brittle or semi-brittle mechanisms of behaviour and failure, on the basis of shear forces anywhere along the length of the member, taking into account the reduction of the cyclic shear force capacity in flexural plastic hinges owing to the inelastic flexural deformations there.

At the “operational” performance level, all members, primary or secondary for earthquake resistance, are required to remain elastic.

The following sections deal with the quantification of the deformation capacities of RC members that enter in the above design and verification criteria (notably \( \delta_{y} \), \( \delta_{uk,0.05} \), \( \delta_{u,m} \) and \( \delta_{u,m-\sigma} \)), as well as of the cyclic shear force capacity in flexural plastic hinges (notably \( VR_{m} \) and \( VR_{m-\sigma} \)), as affected by the magnitude of the inelastic flexural deformations there.

### 3.1.2 Deformations of RC members at yielding

The deformations at member yielding have been selected as the criteria for the “operational” performance level (cf. Table 3.1). The deformation measures of interest are:
- the curvature, \( \varphi \), at the level of the cross-section, and
- the chord rotation, \( \theta \), at the member end, at the member level.

For RC members with rectangular compression zone (typical of building construction)
the values of \( \varphi \) and \( \theta \) at yielding, \( \varphi_y \) and \( \theta_y \), respectively, have been given in Section 2.2.1.3 in terms of the member geometric and reinforcement parameters, based on theoretical considerations, calibrated/modified on the basis of the available test results.

### 3.1.3 Flexure-controlled ultimate deformations under uniaxial loading

#### 3.1.3.1 Uniaxial ultimate curvature of RC members with rectangular compression zone on the basis of first principles

The ultimate curvature, \( \varphi_u \), of RC members is calculated on the basis of the plane sections hypothesis, equilibrium and nonlinear \( \sigma-\varepsilon \) laws. In the present case the laws adopted are:

- For concrete, the \( \sigma-\varepsilon \) law rises parabolically until a strain \( \varepsilon_{co} \) is reached and stays then horizontal up to a strain \( \varepsilon_{cu} \). In the confined core of the section, the values of \( \varepsilon_{co} \) and \( \varepsilon_{cu} \) and of the \( f_c \) concrete strength increase with confinement by stirrups.

- At the relatively low steel strains associated with section ultimate conditions due to concrete crushing, the \( \sigma-\varepsilon \) law of reinforcing steel is elastic-perfectly plastic. At the large strains typical of section failure due to steel rupture the \( \sigma-\varepsilon \) law of rebars is taken elastic-linearly strain-hardening, from its yield stress \( f_y \) at \( \varepsilon_y \) to the ultimate strength \( f_t \) at the “uniform elongation” of \( \varepsilon_{su} \).

If the section fails by steel rupture at elongation \( \varepsilon_{su} \) at a compression zone depth \( \xi_{su} \) (normalized to the effective depth \( d \)), before concrete crushing, the ultimate curvature is:

\[
\varphi_{su} = \frac{\varepsilon_{su}}{(1 - \xi_{su})d} \tag{3.1}
\]

while, if the section fails by crushing of the extreme concrete fibres, at a compression zone depth \( \xi_{cu} \) (normalized to \( d \)), the ultimate curvature is:

\[
\varphi_{cu} = \frac{\varepsilon_{cu}}{\xi_{cu}d} \tag{3.2}
\]

The values of \( \xi_{su}, \xi_{cu} \) are determined from Eqs. (3.8)-(3.13), depending on the value of the distance of extreme tension or compression reinforcement, \( d_t \), normalized to \( d \) as \( \delta_t = d_t/d \), relative to the limits of Eqs. (3.3), (3.4) and of the axial load ratio \( \nu = N/bdf_c \) relative to those of Eqs. (3.5)-(3.7), according to the flow chart of the following two pages (with \( \omega_1 = \rho_1 f_c/f_s \), \( \omega_2 = \rho_2 f_s/f_s \), \( \omega_3 = \rho_v f_s/f_s \) and \( \rho_1, \rho_2, \rho_v \) normalized to \( bd \)).

\[
\delta_t \leq \frac{\varepsilon_{cu} - \varepsilon_y^2}{\varepsilon_{cu} + \varepsilon_{su}} \tag{3.3}
\]
\[
\delta_1 \leq \frac{\epsilon_{cu} - \epsilon_{y2}}{\epsilon_{cu} + \epsilon_{y1}} \\
\frac{\delta_1 \epsilon_{m1} + \epsilon_{y2} - (1 - \delta_1) \epsilon_{m1}}{\epsilon_{m1} + \epsilon_{y2}} + \omega_2 - \omega_1 \frac{f_{y1}}{f_{y2}} = \frac{\omega_2}{(1 - \delta_1) \epsilon_{m1} + \epsilon_{y2}} \left[ \epsilon_{m1} - \epsilon_{y2} + \frac{1}{2} (\epsilon_{m1} - \epsilon_{shw}) \left( 1 + \frac{f_{c}}{f_{y1}} \right) \right] \\
\equiv V_{c,y2} \leq V \leq V_{c,v} = \frac{\epsilon_{cu} - \epsilon_{m1}}{\epsilon_{cu} + \epsilon_{m1}} + \omega_2 - \omega_1 \frac{f_{y1}}{f_{y2}} \left( \frac{\epsilon_{cu} - \epsilon_{m1}}{\epsilon_{cu} + \epsilon_{y1}} - 1 \right) + \delta_1 \frac{\epsilon_{cu} - \epsilon_{co}}{\epsilon_{cu} + \epsilon_{y1}} \\
\omega_2 - \omega_1 + \frac{\omega_2}{1 - \delta_1} \left( \epsilon_{cu} - \epsilon_{y1} - 1 \right) + \frac{\epsilon_{cu} - \epsilon_{co}}{1 - \delta_1} \equiv V_{c,y1} \leq V < \\
\frac{\omega_2}{\epsilon_{y2}} \left( (1 - \delta_1) \frac{\epsilon_{cu} - \epsilon_{y2}}{\epsilon_{cu} - \epsilon_{y1}} - \omega_1 + \frac{\omega_2}{2 \epsilon_{y2}} \left( \epsilon_{cu} - \frac{1 + \delta_1}{1 - \delta_1} \right) + \frac{\epsilon_{cu} - \epsilon_{co}}{1 - \delta_1} \right) \equiv V_{c,y1} \leq V < \\
\frac{-V_{c,y2}}{\omega_2} - \omega_1 \frac{(1 - \delta_1) \epsilon_{cu} - \epsilon_{y2}}{\epsilon_{y1} - \delta_1} + \frac{\omega_2}{\delta_1 \epsilon_{y2}} \left( \frac{1 + \delta_1}{1 - \delta_1} \epsilon_{cu} - \epsilon_{y2} \right) + \frac{\epsilon_{cu} - \epsilon_{co}}{\epsilon_{cu} - \epsilon_{y2}} \\
\approx \frac{c_{su}}{\epsilon_{su}} \approx \left( 1 - \delta_1 \right) \left( 1 + \frac{\epsilon_{co}}{3 \epsilon_{su1}} \right) + \frac{2 + \frac{1}{2} (\epsilon_{shw} \epsilon_{su1})}{1 + \frac{f_{c}}{f_{y2}}} \omega_2 \\
(3.4) \\
(3.5) \\
(3.6) \\
(3.7) \\
(3.8)
\]
Unconfined full section – Steel rupture

\[ \delta_1 \text{ satisfies Eq.}(3.3) \]

\[ \nu < \nu_{c_2} - \text{LHS Eq.}(3.5) \]

\[ \xi_u \text{ from Eq.}(3.9), M_{R_u} \text{ from Eq.}(3.14) \]

\[ \nu < \nu_{c_2} - \text{RHS Eq.}(3.5) \]

\[ \xi_u \text{ from Eq.}(3.8), M_{R_u} \text{ from Eq.}(3.15) \]

Unconfined full section – Spalling of concrete cover

\[ \delta_1 \text{ satisfies Eq.}(3.4) \]

\[ \nu < \nu_{c_1} - \text{LHS Eq.}(3.7) \]

\[ \xi_u \text{ from Eq.}(3.10), M_{R_u} \text{ from Eq.}(3.17) \]

\[ \nu < \nu_{c_1} - \text{RHS Eq.}(3.6) \]

\[ \xi_u \text{ from Eq.}(3.11), M_{R_u} \text{ from Eq.}(3.18) \]

\[ \xi_u \text{ from Eq.}(3.12), M_{R_c} \text{ from Eq.}(3.19) \]

Compute moment resistances:
- \( M_{R_u} \) (of full, unspalled section)
- \( M_{R_c} \) (of confined core, after spalling of cover)

\[ M_{R_u} < 0.8 M_{R_c} ? \]

Ultimate curvature of confined core after spalling of concrete cover

\[ \phi_{cu} \text{ from Eq.}(3.2) \]

Flow chart of next page
Figure 3.1: Flow chart with the steps for the calculation of the ultimate curvature, for the full section before or at spalling of the concrete cover (previous page) and for the confined core of the section after spalling of the concrete cover (this page).
If spalling of the concrete cover is attained without prior steel rupture and the section has a well-confined concrete core, the subsequent moment resistance of the confined core, $M_{Ro}$, may reach and exceed 80% of the moment resistance of the unconfined full section, $M_{Rc}$. Then the section will reach the ultimate condition in flexure (conventionally taken to occur when the resistance cannot increase above 80% of the maximum ever resistance previously attained) after spalling. Then the 2nd page of the flow chart in Figure 3.1 applies, with the confined core considered as the section (its dimensions denoted by an asterisk) and with the properties of the confined concrete, $f_{cc}$, $\epsilon_{cc}$, used in lieu of $f_{cu}$, $\epsilon_{cu}$.

These properties are given in the next section, as a function of confinement.

The moment resistance of the full section or of the confined core can be computed from Eqs. (3.14)-(3.19) below according to Flow Chart 3.1.
\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{6(1 - \bar{\xi}^2)} \left[ 1 - \bar{\xi} + \bar{\xi} \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \right]
\]

\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{1 - \bar{\xi}^2} \left[ (\bar{\xi} - \bar{\xi}) (1 - \bar{\xi}) - \frac{1}{3} \left( \frac{1 - \bar{\xi}}{E_{m1}} \right) \right]
\]

\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{4(1 - \bar{\xi}^2)} \left[ 1 - \bar{\xi} + \bar{\xi} \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \right]
\]

\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{1 - \bar{\xi}^2} \left[ (\bar{\xi} - \bar{\xi}) (1 - \bar{\xi}) - \frac{1}{3} \left( \frac{1 - \bar{\xi}}{E_{m1}} \right) \right]
\]

\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{4(1 - \bar{\xi}^2)} \left[ 1 - \bar{\xi} + \bar{\xi} \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{E_{m1}} \right) \left( 1 - \frac{E_{cm}}{E_{m2}} \right) \left( 1 - \bar{\xi} \right) \right]
\]

\[
\frac{M_{Rc}}{bd^2 f_c} = \left(1 - \bar{\xi}^2 \right) \left[ \frac{\bar{\xi}}{2} \left( 1 - \bar{\xi} \right) \left( 1 - \frac{E_{cm}}{3E_{m1}} \right) \left( \frac{E_{cm}}{E_{m1}} + \frac{\Delta e_{ml}}{4E_{m1}} \left( 1 - \bar{\xi} \right) \right) \right] + \frac{1 - \bar{\xi}}{2} \left( \frac{\omega_{1}}{f_{y1}} + \omega_{2} \frac{\xi - \bar{\xi}}{E_{m2}} \right) + \frac{\omega_{v}}{1 - \bar{\xi}^2} \left[ (\bar{\xi} - \bar{\xi}) (1 - \bar{\xi}) - \frac{1}{3} \left( \frac{1 - \bar{\xi}}{E_{m1}} \right) \right]
\]
3.1.3.2 Ultimate strains of steel and concrete at section ultimate curvature

Optimal fit of the ultimate curvature values, $\rho_u$, from Section 3.1.3.1 to experimental curvatures in RC members with rectangular compression zone (typical of building construction) at flexure-controlled failure under monotonic or cyclic loading is achieved, if the material parameters are chosen as follows:

Confined concrete strength $f_{cc}$, according to [Newman and Newman 1971], [CEN 2005a]:

$$f_{cc} = f_c \left( 1 + 3.7 \left( \frac{\alpha \rho_s f_{yw}}{f_c} \right)^{0.86} \right)$$

(3.20)

where $\rho_s$ is the transverse reinforcement ratio (minimum among the two transverse directions), $f_{yw}$ its yield stress and $\alpha$ the confinement effectiveness factor [according to Sheikh and Uzumeri, 1982, CEN 2004a, CEN 2005a]:

$$\alpha = \left( 1 - \frac{s_h}{2b_c} \right) \left( 1 - \frac{s_h}{2h_c} \right) \left( 1 - \sum b_i^{2} \right)$$

(3.21)

with $s_h$ being the centreline spacing of stirrups, $b_c$ and $h_c$ the confined core dimensions to the centreline of the hoop and $b_i$ the centreline spacing along the section perimeter of the longitudinal bars (indexed by $i$) that are laterally engaged by a stirrup corner or a cross-tie.

Ultimate strain of concrete, including the effect of any confinement and a dependence on size (he is the depth of the confined core, or the full section depth in the case of spalling of the extreme compression fibres):

- for monotonic loading:
  $$\varepsilon_{cu,c} = 0.0035 + \left( \frac{10}{h_c} \right)^2 + 0.57 \cdot \left( \alpha \rho_s f_{yw} / f_{cc} \right)$$

(3.22a)

- for cyclic loading:
  $$\varepsilon_{cu,c} = 0.0035 + \left( \frac{10}{h_c} \right)^2 + 0.4 \cdot \left( \alpha \rho_s f_{yw} / f_{cc} \right)$$

(3.22b)

A limit on the available elongation of the tension reinforcement:
− for monotonic loading: \[ \varepsilon_{su} = \frac{7}{12} \varepsilon_{su,\text{nominal}} \] \hspace{1cm} (3.23a)

− for cyclic loading: \[ \varepsilon_{su} = \frac{3}{8} \varepsilon_{su,\text{nominal}} \] \hspace{1cm} (3.23b)

where \( \varepsilon_{su,\text{nominal}} \) is the uniform elongation at tensile strength in the standard monotonic test of steel coupons.

The “fixed-end rotation” due to slippage of bars from their anchorage zone beyond the section of maximum moment at ultimate conditions of that section may be taken according to a fitting of an extension of Eq. (2.19) in Section 2.2.1.3 to experimentally measured or derived values (e.g., from measurements at different gauge length in the same test) of such “fixed-end rotation” at ultimate curvature of that section:

\[ \theta_{u,\text{slip}} = \frac{\varphi_u d_{bl,\text{f}_y}}{8 \sqrt{f_c}} (f_y \text{ and } f_c \text{ in MPa}). \] \hspace{1cm} (3.24a)

\[ \theta_{u,\text{slip}} = \frac{\varphi_u d_{bl,\text{f}_y}}{16 \sqrt{f_c}} (f_y \text{ and } f_c \text{ in MPa}). \] \hspace{1cm} (3.24b)

where \( d_{bl} \) is the mean diameter of longitudinal bars and \( \varphi_u \) the ultimate curvature.

<table>
<thead>
<tr>
<th>( \frac{\varphi_{u,\text{exp}}}{\varphi_{u,\text{Sections 3.1.3.1, 3.1.3.2}}} \text{ for different testing and failure modes} )</th>
<th># of data</th>
<th>mean*</th>
<th>median*</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All tests</td>
<td>454</td>
<td>1.115</td>
<td>1.005</td>
<td>49.3%</td>
</tr>
<tr>
<td>Monotonic tests</td>
<td>257</td>
<td>1.14</td>
<td>1.02</td>
<td>53.4%</td>
</tr>
<tr>
<td>Cyclic tests</td>
<td>197</td>
<td>1.085</td>
<td>0.99</td>
<td>42.6%</td>
</tr>
<tr>
<td>w/o slippage of bars from the anchorage zone</td>
<td>333</td>
<td>1.15</td>
<td>1.01</td>
<td>50.5%</td>
</tr>
<tr>
<td>w/ slippage of bars from the anchorage zone</td>
<td>121</td>
<td>1.025</td>
<td>0.99</td>
<td>44.0%</td>
</tr>
<tr>
<td>Failure at spalling of full section</td>
<td>53</td>
<td>1.20</td>
<td>1.00</td>
<td>56.5%</td>
</tr>
<tr>
<td>Failure due to crushing of confined core — monotonic tests</td>
<td>105</td>
<td>1.08</td>
<td>1.02</td>
<td>51.9%</td>
</tr>
<tr>
<td>Failure due to crushing of confined core — cyclic tests</td>
<td>80</td>
<td>1.17</td>
<td>1.01</td>
<td>48.6%</td>
</tr>
<tr>
<td>Failure due to steel rupture — monotonic tests</td>
<td>115</td>
<td>1.13</td>
<td>0.99</td>
<td>52.3%</td>
</tr>
<tr>
<td>Failure due to steel rupture — cyclic tests</td>
<td>101</td>
<td>1.05</td>
<td>1.01</td>
<td>35.7%</td>
</tr>
</tbody>
</table>
3.1.3.3 Uniaxial ultimate chord rotation from the plastic hinge length

If inelastic behaviour and failure is controlled by flexure, the familiar description of the plastic component of chord rotation over the shear span \( L_s \) as the product of the plastic component of ultimate curvature, \( \phi_u - \phi_y \), and a plastic-hinge length, \( L_{pl} \), recognizing the contribution of fixed-end rotation due to bar pull-out from the anchorage zone beyond the end of the member where failure takes place, gives for the ultimate chord rotation:

\[
\theta_u = \theta_y + a_{sl}(\theta_{u,slip} - \theta_{y,slip}) + (\phi_u - \phi_y)L_{pl}\left(1 - \frac{L_{pl}}{2L_s}\right) \tag{3.25}
\]

where the 1st term is the chord rotation at yielding from Eqs.(2.20) in Section 2.2.1.3, the 2nd term is the additional (from yielding to ultimate) fixed-end rotation due to bar pull-out from the anchorage zone from Eqs.(2.19) and (3.24), with \( a_{sl} = 0 \) if slippage of longitudinal bars from the anchorage zone is not physically possible and with \( a_{sl} = 1 \) if it is, and the 3rd term is the plastic deformation of the flexural plastic hinge.

Empirical expressions for \( L_{pl} \) in Eq.(3.25) cannot be developed independently of the models used for \( \theta_y, \theta_{u,slip}, \theta_{u,slip}, \phi_u \) and \( \phi_y \). Considering the comparisons with experimental data in Tables 2.4 and 3.1 as a confirmation of the models in Section 2.2.1.3 for \( \phi_y, \theta_y, \theta_{u,slip} \), and in Sections 3.1.3.1, 3.1.3.2 for \( \theta_{u,slip}, \phi_u \) and for the material parameters (including those for confined concrete), empirical expressions have been derived for the plastic hinge length, \( L_{pl} \), to fit Eq.(3.25) to the data on ultimate chord rotation, \( \theta_u \), of uniaxial tests on members failing in flexure, using the values of \( \theta_y, \theta_{u,slip}, \theta_{u,slip}, \phi_u \) and \( \phi_y \) derived from Sections 2.2.1.3 and 3.1.3.1, 3.1.3.2. Note that the same expression for \( L_{pl} \) cannot fit both the monotonic and the cyclic data. Eq. (3.26) were found to provide the best overall fit to \( \theta_u \) for beams and rectangular columns or walls

- for monotonic loading: \( L_{pl} = 0.04L_s + 1.2h \) \hspace{1cm} (3.26a)
- for cyclic loading: \( L_{pl} = 0.09L_s + 0.2h \) \hspace{1cm} (3.26b)

Table 3.2 (rows 1 to 6) gives statistics of the ratio of experimental - ultimate chord rotations to those predicted according to the present section.

3.1.3.4 Empirical uniaxial ultimate chord rotation of RC members

As the fitting to the test results on uniaxial flexure-controlled ultimate chord rotation by
the Section 3.1.3.3 model, although optimal, is not satisfactory, two empirical alternatives have been developed. The first one is for the total ultimate chord rotation, $\theta_u$; the other is for its plastic component, $\theta_{upl} = \theta_u - \theta_y$, with the elastic component, $\theta_y$, given by Eq.(2.20):

$$\theta_u = a_y (1-0.43 a_y) \left(1+\frac{a_{ax}}{2}\right) \left(1-\frac{3}{8} a_{awd}\right) \left[\max\left(\frac{0.01, \omega_1}{\max(0.01, \omega)}\right)\right]^{0.225} \left(\frac{L_s}{h}\right)^{0.35} \left[\max\left(\frac{0.01, \omega_2}{\max(0.01, \omega)}\right)\right]^{0.3} \left(\frac{L_a}{h}\right)^{0.35} \frac{f_{yw}}{f_c} 1.25^{100\rho_d \sqrt{\nu/\omega}}$$ (3.27a)

$$\theta_{upl} = a_y (1-0.52 a_y) (1+a_{ax}/1.6)(1-0.43 a_{awd}) \left[\max\left(\frac{0.01, \omega_1}{\max(0.01, \omega)}\right)\right]^{0.3} \left(\frac{L_s}{h}\right)^{0.35} \left(\frac{L_a}{h}\right)^{0.35} \frac{f_{yw}}{f_c} 1.275^{100\rho_d \sqrt{\nu/\omega}}$$ (3.27b)

where:

- $a_y$ and $a_{awd}$: coefficients for the type of steel, equal to $a_y = 0.0185$ and $a_{awd} = 0.0185$ for ductile hot-rolled or for heat-treated (tempcore) steel and to $a_y = 0.0115$ and $a_{awd} = 0.009$ for cold-worked steel;
- $a_{ax}$: zero-one variable for the type of loading, equal to 0 for monotonic loading and to 1 for cyclic loading;
- $a_{ax}$: zero-one variable for slip, equal to 1 if there can be slip of the longitudinal bars from their anchorage beyond the section of maximum moment, or to 0 if not (cf. Eq. (3.25));
- $a_{wall}$: zero-one variable for walls, equal to 1 for shear walls and to 0 for beam/columns;
- $\nu=N/bhf_c$ (with $b=width$ of compression zone, $N=axial$ force, positive for compression);
- $\omega_1$: mechanical reinforcement ratio of tension and “web” longitudinal reinforcement, $(\rho_1 f_y + \rho_1 f_y)/f_c$;
- $\omega_2$: mechanical reinforcement ratio of compression longitudinal reinforcement, $\rho_2 f_c/f_c$;
- $f_c$: uniaxial (cylindrical) concrete strength (MPa)
- $L_s/h=M/Vh$: shear span ratio at the section of maximum moment;
- $\rho_d = A_{sh}/b_{sh}h$: ratio of transverse steel parallel to the direction of loading;
- $f_{yw}$: yield stress of transverse steel;
- $\alpha$: confinement effectiveness factor from Eq.(3.21) [Sheikh and Uzumeri, 1982];
- $\rho_d$: steel ratio of diagonal reinforcement in each diagonal direction.

The statistics of the fitting listed in rows 7 to 18 of Table 3.2 show that Eqs. (3.27a) and (3.27b) are practically equivalent in terms of predictive ability, while both are better than the model in Sections 3.1.3.1-3.1.3.3.

### 3.1.4 Flexure-controlled ultimate deformations under biaxial loading

The few available biaxial (bidirectional) tests on flexure-controlled RC columns suggest that, at ultimate deformation, the chord rotation components parallel to the sides of the
cross-section, $\theta_{uy}$ and $\theta_{uz}$, have an average margin between 5 and 17.5% beyond a circular interaction diagramme of the form:

$$
\left( \frac{\theta_{uy}}{\theta_{uy,uni}} \right)^2 + \left( \frac{\theta_{uz}}{\theta_{uz,uni}} \right)^2 = 1
$$

(3.28)

in which the ultimate chord rotations under unidirectional bending parallel to the sides of the section, $\theta_{uy,uni}$ and $\theta_{uz,uni}$, are computed either through the procedure of Sections 3.1.3.1-3.1.3.3 based on curvatures and the plastic hinge length, or by means of the purely empirical expressions statistically fitted to the uniaxial data in Section 3.1.3.4. The statistics of the fitting in Table 3.3 suggest that the alternative uniaxial models are practically equivalent for use in Eq. (3.28). The smaller scatter about the mean compared to the uniaxial data in Table 3.3 is due to the very small sample size of the biaxial data.

<table>
<thead>
<tr>
<th>$\theta_{u,exp} / \theta_{u,predicted}$ for different models and test conditions</th>
<th># of data</th>
<th>mean*</th>
<th>median*</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Sections 3.1.3.1-3.1.3.3} - All tests$</td>
<td>1307</td>
<td>1.105</td>
<td>0.995</td>
<td>53.6%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Sections 3.1.3.1-3.1.3.3} - All monotonic tests$</td>
<td>295</td>
<td>1.19</td>
<td>1.00</td>
<td>67.4%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Sections 3.1.3.1-3.1.3.3} - All cyclic tests$</td>
<td>1012</td>
<td>1.075</td>
<td>0.995</td>
<td>47.4%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Sections 3.1.3.1-3.1.3.3} - w/o slippage of bars from anchorage$</td>
<td>211</td>
<td>1.165</td>
<td>0.96</td>
<td>65.1%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Sections 3.1.3.1-3.1.3.3} - w/ slippage of bars from anchorage$</td>
<td>1096</td>
<td>1.095</td>
<td>1.005</td>
<td>50.8%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - walls w/ bar slippage from anchorage$</td>
<td>78</td>
<td>1.175</td>
<td>1.005</td>
<td>61.1%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - All tests$</td>
<td>1307</td>
<td>1.05</td>
<td>0.995</td>
<td>42.8%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - All monotonic tests$</td>
<td>295</td>
<td>1.14</td>
<td>1.00</td>
<td>53.6%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - All cyclic tests$</td>
<td>1012</td>
<td>1.025</td>
<td>0.99</td>
<td>37.8%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - w/o slippage of bars from anchorage$</td>
<td>211</td>
<td>1.10</td>
<td>0.98</td>
<td>50.2%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - w/ slippage of bars from anchorage$</td>
<td>1096</td>
<td>1.045</td>
<td>0.995</td>
<td>40.9%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27a)} - Walls w/ slippage of bars from anchorage$</td>
<td>78</td>
<td>0.985</td>
<td>0.995</td>
<td>31.9%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27b)} - All tests$</td>
<td>1307</td>
<td>1.05</td>
<td>0.995</td>
<td>42.7%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27b)} - All monotonic tests$</td>
<td>295</td>
<td>1.13</td>
<td>1.00</td>
<td>53.7%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27b)} - All cyclic tests$</td>
<td>1012</td>
<td>1.03</td>
<td>0.99</td>
<td>37.7%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27b)} - w/o slippage of bars from anchorage$</td>
<td>211</td>
<td>1.12</td>
<td>0.995</td>
<td>50.5%</td>
</tr>
<tr>
<td>$\theta_{u,exp} / \theta_{u,Eq. (3.27b)} - w/ slippage of bars from anchorage$</td>
<td>1096</td>
<td>1.04</td>
<td>0.995</td>
<td>40.6%</td>
</tr>
</tbody>
</table>
\[ \frac{\theta_{\text{exp}}}{\theta_{\text{u, Eq. (3.27b)}}} \] – Walls w/ slippage of bars from anchorage

<table>
<thead>
<tr>
<th>Quantity at ultimate under bidirectional loading</th>
<th># of data</th>
<th>mean</th>
<th>median</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRSS of ( \theta_{\text{uy,exp}}/\theta_{\text{uy, Sect. 3.1.3.3}} ) &amp; ( \theta_{\text{uz,exp}}/\theta_{\text{uz, Sect. 3.1.3.3}} )</td>
<td>36</td>
<td>1.10</td>
<td>1.05</td>
<td>28.7%</td>
</tr>
<tr>
<td>SRSS of ( \theta_{\text{uy,exp}}/\theta_{\text{uy, Eq.(3.27a)}} ) &amp; ( \theta_{\text{uz,exp}}/\theta_{\text{uz, Eq.(3.27a)}} )</td>
<td>36</td>
<td>1.27</td>
<td>1.175</td>
<td>23.2%</td>
</tr>
<tr>
<td>SRSS of ( \theta_{\text{uy,exp}}/\theta_{\text{uy, Eq.(3.27b)}} ) &amp; ( \theta_{\text{uz,exp}}/\theta_{\text{uz, Eq.(3.27b)}} )</td>
<td>36</td>
<td>1.25</td>
<td>1.15</td>
<td>23.3%</td>
</tr>
</tbody>
</table>

3.1.5 Acceptable ultimate deformations for RC members under uni- or bi-directional cyclic loading, at different performance levels

In this section, the outcome of Sections 3.1.3 and 3.1.4 is used to derive mean, mean-minus-standard deviation and 5%-fractile value of ultimate deformations of RC members for use in the design and verification criteria of Table 3.1.

The most common and appropriate deformation measure is the chord rotation, \( \theta \), at member ends. Performance evaluation of RC members under biaxial loading on the basis of ultimate chord rotations through the circular interaction diagram of Eq. (3.28), which includes uniaxial loading as a special case, is, on average, on the safe side by 5-17.5%. The lower scatter about the mean compared to that of the uniaxial is due to the small sample size of the biaxial data. It is proposed, therefore, to use the same acceptable deformations in biaxial loading as for unidirectional loading.

If the model in Section 3.1.3.3 is used to calculate the ultimate chord rotation on the basis of the plastic hinge length (with curvature, \( \varphi \), chord rotation, \( \theta \), and fixed-end rotation, \( \theta_{\text{slip}} \), at yielding according to Section 2.2.1.3, and curvature, \( \varphi \), and fixed-end rotation, \( \theta_{\text{slip}} \), at ultimate according to Sections 3.1.3.1, 3.1.3.2), the ratio of experimental-to-predicted ultimate chord rotation in the 1012 cyclic tests has a 5%-fractile of 45% and a coefficient of variation of 47.4%. Therefore:
- the outcome of the model in Section 3.1.3.3 is the mean ultimate chord rotation, \( \theta_{\text{um}} \);
- for uniaxial verifications \( \theta_{\text{um}} - \sigma = 0.525 \theta_{\text{um}} \) and for biaxial ones the mean-minus-standard deviation value of the right-hand-side of Eq. (3.28) may be taken equal to 0.525;
- for uniaxial verifications \( \theta_{\text{um}}(0.05) = 0.45 \theta_{\text{um}} \) and for biaxial ones the 5%-fractile value of the right-hand-side of Eq. (3.28) may be taken equal to 0.45.

See footnote of Table 2.4 for the median vs the mean for large sample size.
If Eqs. (3.27a) or (3.27b) (with Eq. (3.27b) supplemented with $\theta_y$ from Section 2.2.1.3) are used to calculate the ultimate chord rotation, the ratio of experimental-to-predicted ultimate chord rotation in the 1012 cyclic tests has a 5%-fractile of 49% or 50%, respectively, and a coefficient of variation of 37.8% or 37.7%, respectively. Therefore:

- the outcome of the model in Section 3.1.3.3 is the mean ultimate chord rotation, $\theta_{um}$;
- for uniaxial verifications $\theta_{um}=0.625\theta_{um}$ and for biaxial ones the mean-minus-standard deviation value of the right-hand-side of Eq. (3.28) may be taken equal to 0.625.

- for uniaxial verifications $\theta_{uk,0.05}=0.5\theta_{um}$ and for biaxial ones the 5%-fractile value of the right-hand-side of Eq. (3.28) may be taken equal to 0.5.

If the design and verification criteria of Table 3.1 are applied using the curvature at the member end sections as deformation measure, and the model in Sections 3.1.3.1, 3.1.3.2 are used to calculate the ultimate curvature, $\varphi_u$, the ratio of experimental-to-predicted ultimate curvature in the just 197 cyclic tests in which curvature measurements are available has a 5%-fractile of 50% and a coefficient of variation of 42.6%. Therefore, with the qualification of the limited amount of cyclic uniaxial data on curvatures and the complete lack of biaxial ones to support extension of Eq. (3.28) to ultimate curvatures:

- the outcome of the model in Sections 3.1.3.1, 3.1.3.2 is the mean ultimate curvature, $\varphi_{um}$;
- for uniaxial verifications $\varphi_{um}=0.575\varphi_{um}$ and for biaxial ones the mean-minus-standard deviation value of the right-hand-side of Eq. (3.28) (extended to ultimate curvatures) may be taken equal to 0.575;
- for uniaxial verifications $\varphi_{uk,0.05}=0.5\varphi_{um}$ and for biaxial ones the 5%-fractile value of the right-hand-side of Eq. (3.28) (extended to ultimate curvatures) may be taken equal to 0.5.

### 3.1.6 Shear resistance in diagonal tension under inelastic cyclic deformations after flexural yielding

If it takes place before flexural yielding, ultimate failure of concrete members in shear occurs at relatively low deformations and is considered as “brittle” failure. Sometimes concrete members that yield first in flexure, may ultimately fail in the flexural plastic hinge under cyclic loading with the failure mode showing strong and clear effects of shear (inclined cracks increase in width and extend with cycling despite the gradual drop of peak force resistance with load cycling). At the same time, phenomena normally associated with flexural failure, such as a single wide crack transverse to the axis at the section of maximum moment, disintegration of the compression zone and/or buckling of longitudinal bars next to the section of maximum moment, are not so pronounced at failure. By contrast, in a flexure-controlled failure these latter phenomena govern, often
leading to rupture of a longitudinal bar, whereas the width of any inclined cracks that may have formed at the beginning decreases and such cracks may even disappear, as the peak force resistance drops with load cycling after the flexure-controlled ultimate strength.

Failure in shear under cyclic loading, after initial flexural yielding is termed “ductile shear” failure [Kowalsky and Priestley, 2000]. It is normally associated with diagonal tension and yielding of web reinforcement, rather than by web crushing. It has by now prevailed to quantify this failure mode via a shear resistance $V_R$, (as this is controlled by web reinforcement according to the well-established Mörsch truss analogy) that decreases with the (displacement) ductility ratio under cyclic loading [Ascheim and Moehle, 1992, Kowalsky and Priestley, 2000, Moehle et al, 2001]. As the number of available cyclic tests that led to “ductile shear” failure is not sufficient to support development of an independent (statistical or mechanical) model for the deformation capacity of concrete members as affected or controlled by shear, this work has also adopted the solid basis of the Mörsch analogy for shear, to describe in force terms a failure mode which is controlled by deformations.

A large data set of columns and walls ultimately failing by “ductile shear” under cyclic loading has been used to develop two models for the shear resistance, $V_R$, as a function of the plastic chord rotation ductility ratio, $\mu_{pl}$ (ratio of post-elastic chord rotation at “ductile shear” failure, to chord rotation at yielding, $\theta_y$, from Eqs.(2.19)). In both models the effect of axial force, $N$, on $V_R$ is accounted for through a separate term, representing the contribution to shear resistance of the transverse to the member axis component of the compression strut between the two ends of the member, as in the CEB/FIP Model Code 90, in EN 1992-1-1 [CEN 2005c] and [Kowalsky and Priestley, 2000]. A 45° truss inclination is considered, as in [Moehle et al, 2001], because truss inclinations other than 45° are normally taken when only the web reinforcement is considered to contribute to $V_R$ ($V_w$ term), without a separate concrete contribution ($V_c$ term).

$$V_s = \frac{h-x}{2L_c} \min(N, 0.5A_f f_c) + 0.16 \left(1 - 0.095 \min(5, \mu_{pl}) \max(0.5, 100\rho_{wc}) \left(1 - 0.16 \min\left(\frac{5}{h}, \frac{L_c}{x} \right) \sqrt{f_c A_i + V_c}\right) \right)$$

$$V_s = \frac{h-x}{2L_c} \min(N, 0.5A_f f_c) + 0.05 \min(5, \mu_{pl}) \max(0.5, 100\rho_{wc}) \left(1 - 0.16 \min\left(\frac{5}{h}, \frac{L_c}{x} \right) \sqrt{f_c A_i + V_c}\right)$$

where:
- $h$: depth of cross-section;
- $x$: compression zone depth;
- $N$: compressive axial force (positive, taken as zero for tension);
\( L_s/h = M/Vh \): shear span ratio at member end;

- \( A_c \): cross-section area, equal to \( b_w d \) for cross-sections with rectangular web of width (thickness) \( b_w \) and structural depth \( d \);
- \( f_c \): concrete strength (MPa);
- \( \rho_{\text{tot}} \): total longitudinal reinforcement ratio;
- \( V_w \): contribution of transverse reinforcement to shear resistance, taken equal to:

\[
V_w = \rho_w b_w z f_{yw} 
\]

(3.30)

where:
- \( b_w \) is the width (thickness) of the rectangular web,
- \( \rho_w \) and \( f_{yw} \) are the ratio and yield stress of transverse reinforcement, and
- \( z \) is the internal lever arm (\( z = d - d_1 \) in beam/columns, \( z = 0.8h \) in rectangular walls).

Table 3.4 (rows 1 and 2) gives statistics of the ratio of the experimental to the so-predicted shear resistance.

<table>
<thead>
<tr>
<th>Shear resistance for different failure modes</th>
<th># of data</th>
<th>mean*</th>
<th>median*</th>
<th>coef. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{R,\text{exp}} )/( V_{R,\text{eq. (3.29a)}} ) – beams/columns, diagonal tension failure</td>
<td>193</td>
<td>0.99</td>
<td>0.985</td>
<td>14.6%</td>
</tr>
<tr>
<td>( V_{R,\text{exp}} )/( V_{R,\text{eq. (3.29b)}} ) – beams/columns, diagonal tension failure</td>
<td>193</td>
<td>0.98</td>
<td>0.98</td>
<td>13.3%</td>
</tr>
<tr>
<td>( V_{R,\text{exp}} )/( V_{R,\text{eq. (3.31)}} ) – walls, web diagonal compression failure</td>
<td>51</td>
<td>1.035</td>
<td>1.01</td>
<td>17.2%</td>
</tr>
<tr>
<td>( V_{R,\text{exp}} )/( V_{R,\text{eq. (3.32)}} ) – squat columns, diagonal compression failure</td>
<td>37</td>
<td>1.005</td>
<td>1.005</td>
<td>8.9%</td>
</tr>
</tbody>
</table>

*See footnote of Table 2.4 for the median vs the mean for large sample size.

3.1.7 Shear resistance of walls or squat columns in diagonal compression under cyclic deformations

The walls in the database considered to have failed in shear after having yielded in flexure did so by web crushing at a shear force normally less than the predictions of Eqs.(3.29) and a chord rotation much less than the one corresponding to flexure-controlled failure according to Sections 3.1.3.3 or 3.1.3.4. The shear resistance of these walls, as well as of a few cyclically loaded others that experienced web crushing before yielding in flexure (i.e., at \( \mu^\theta_0 = 0 \)) decreases with ductility demand, \( \mu^\theta_0 \), according to the expression:
\[ V_{R,\text{max}} = 0.8(1 - 0.06 \min \{ 5; \mu^p \}) \left( 1 + 1.8 \min \{ 0.15; N/A_s f_c \} \right) \left( 1 + 0.25 \max \{ 0.75; 100 \rho_{\text{ew}} \} \right) \left( 1 - 0.2 \min \{ 2; \frac{L_s}{h} \} \right) \left( \min \{ 100; f_y \} \right) h_s z \] (3.31)

A similar expression was fitted to the test results on squat columns with shear span ratio, \( L_s/h \), less than or equal to 2, failing by web crushing along the diagonal of the column under cyclic loading after flexural yielding:

\[ V_{R,\text{max}} = \frac{4}{7} \left( 1 - 0.02 \min \{ 5; \mu^p \} \right) \left( 1 + 1.35 \frac{N}{A_s f_c} \right) \left( 1 + 0.45(100 \rho_{\text{ew}}) \right) \sqrt{(40MPa; f_y)} h_s z \sin 2\delta \] (3.32)

where \( \delta \) is angle between the diagonal of the column in elevation and the column axis:

\[ \tan \delta = \frac{h}{2L_s} \] (3.33)

Table 3.4 gives statistics of the ratio of experimental-to-predicted shear resistance for these walls and squat columns failing by diagonal compression under cyclic loading.

### 3.1.8 Acceptable shear resistance under inelastic cyclic deformations after flexural yielding, for different performance levels

The shear resistance under inelastic cyclic deformations, \( V_R \) from Eqs. (3.29), (3.30) for diagonal tension failure after flexural yielding, or \( V_{R,\text{max}} \) from Eqs. (3.31)-(3.33) for diagonal compression failure, in all cases as a function of the plastic chord rotation ductility ratio, \( \mu^p \), is taken as the mean value of the cyclic shear resistance \( V_{R,m} \) for the purposes of the shear design and verification criteria of Table 3.1.

The mean-minus-standard deviation value of the cyclic shear resistance may be taken as \( V_{R,m-\sigma} = 0.85 V_{R,m} \) for diagonal tension failure and for diagonal compression failure of walls, or as \( V_{R,m-\sigma} = 0.91 V_{R,m} \) for squat columns in diagonal compression.

The 5%-fractile (lower characteristic value) of the cyclic shear resistance may be taken as \( V_{R,k,0.05} = 0.75 V_{R,m} \) for diagonal tension failure, or as \( V_{R,k,0.05} = 0.8 V_{R,m} \) for diagonal compression failure of walls or squat columns.
Part II: Displacement-Based Design of Bridges
1 DISPLACEMENT-BASED DESIGN METHODOLOGIES FOR BRIDGES

1.1 EVALUATION OF ITERATIVE DISPLACEMENT-BASED DESIGN PROCEDURES FOR BRIDGES

1.1.1 Introduction

This Chapter presents an evaluation of the different approximate methodologies proposed to date for the assessment of the performance of a structure of existing bridges and for the design of new ones, and proposes a set of improvements to guarantee their validity for successful application in practice.

Based on previous work developed by the authors, the premise of this investigation is that regardless of the approximations involved in the different methods considered, the approach used for the evaluation and the design of structures should be only one, which, for the evaluation process considers as known the design of the structure and the seismic demand for which it needs to be evaluated, and as unknown the performance of the structure under design actions, while for the design process considers as known the target performance levels and the seismic demand and, as unknown the design parameters which guarantee such performance levels. With this premise in mind, a critical review of a particular class of approximate linear and non-linear evaluation/design methods based on displacements is carried out, and new alternatives which correct some of the deficiencies of the methods currently used are proposed.

Two methods are considered in this investigation, which have as theoretical foundation the concepts of structural dynamics applied to systems with nonlinear behaviour. In the first method considered, the original structure is substituted by a reference linear elastic structure with elements with reduced stiffness and energy dissipation characteristics consistent with the obtained/expected performance levels. This method, iterative in nature, involves the reduction of a substitute structure (see Section 1.1.3 of the present Part II) to an, incorrectly termed, “equivalent” SDOF system from the performance of which the evaluation or the design conditions for the complete structure may be found. The second method, also iterative, is non-linear in nature and considers as basic assumption that the performance of the complete structure, generally expressed in terms of a modal index, e.g., modal ductility, may be approximately related to that of a reference non-linear SDOF system with a response curve directly derived from the non-linear
capacity of the structure.

To validate the correctness and illustrate the application of both methods, six different bridges, considering regular and irregular cases, are evaluated. The results obtained show that under certain circumstances, both approximate methods fail to give acceptable results, suggesting that there exists a regularity condition, not only related to geometric considerations but also to more general structural and seismic demand characteristics, for which the results from the evaluation or design process may be erroneous. The work done has tried with limited success to eliminate this drawback; nevertheless this problem, still under investigation, persists.

1.1.2 State-of-the-art

Since the early 1990s it has been recognized by Moehle [1992] and Priestley [1993] that current methodologies of earthquake resistant design of structures based on forces and strengths do not agree with the seismic performance observed in real reinforced concrete structures, and that it would be much better to use design methodologies based directly on displacements and deformations and/or other valid seismic performance indices. In accordance with this position, in recent years there have been significant advances in the development of design procedures based on performance having as main objective their incorporation in future design codes. In this context, Moehle [1992] proposed a general framework for earthquake resistant design of structures based on drift displacements with the seismic demand given by displacement response spectra.

The procedure for the displacement based design of SDOF systems or systems which may be reduced to “equivalent” linear SDOF systems, such as those proposed by Priestley [1993], Kowalsky et al [1995], Priestley [2000] and Kowalsky [2002], starts from a target design displacement, based on a deformation capacity guaranteed by an appropriate detailing of the structure. Assuming that reasonable values for the yielding displacements may be estimated from the geometry of the elements, displacement ductility demands may be directly calculated from target peak displacements. Starting with these ductilities and with a set of response displacement spectra, the effective period of an equivalent linear viscoelastic SDOF system is determined at peak displacement, considering an equivalent damping ratio which accounts for the inherent viscous damping characteristics of the structure and that required to consider the energy dissipated by the system through non-linear hysteretic behaviour. The final result of this process is the required yielding strength determined from the peak displacement and the secant stiffness corresponding to the effective period. Calvi and Kingsley [1995] extended this methodology to Multiple Degree Of Freedom (MDOF) structures which may be transformed to an equivalent SDOF system using an assumed deformed configuration of the structure. For buildings it is proposed that the assumed deformed configuration is that corresponding to a
predefined plastic mechanism. The final result of this method is the required strength that should be given to the structure to attain the objective performance.

An alternative approach to the performance based evaluation/design of structures is based on the use of nonlinear static analysis procedures (pushover like analyses) to include, in a simple way, the most important features which influence performance [Freeman, 1978], [Fajfar, 1999]. Examples of the methods which use a single mode approximation are described in documents such as ATC-40 [ATC, 1996] and Eurocode 8 [CEN, 2004].

An improvement of the single mode approximation is to include the contribution of higher modes into the forces used for pushover. Relevant formulations of this multimode approximation are in De Rue [1998], Requena and Ayala [2000] and Gupta and Kunnath [2000]. Even though the application of modal analysis in the inelastic domain to define the distribution of lateral forces used to determine the capacity curve of the structure is theoretically incorrect, the reported results from this approach show an acceptable approximation.

Two more recent approximate methods based on the combination of modal responses are the modal pushover analysis originally proposed by Paret et al. [1996] and later on improved and successfully used by Chopra and Goel [2002] and the Incremental Response Spectrum Analysis (IRSA) proposed by Aydinoğlu [2003]. In the first method, pushover analyses are independently carried out separately for each participating mode and the performance of the structure is obtained by adding the modal contributions using a modal combination rule. All the important modes, identified in the initial - elastic state, are used separately to determine the distribution of forces for the pushover analyses, i.e., the number of analyses is equal to the number of important modes in the elastic state.

The IRSA method was originally developed as an approximate step-by-step piecewise dynamic modal analysis for non-linear structures and then conveniently simplified for practical applications using smooth response spectrum [Aydinoğlu, 2003]. The method takes into account the influence of all important modes and the changes in the dynamic properties of the structure every time a plastic hinge occurs. Modal capacity diagrams for each important mode are constructed through modal analyses. To calculate the performance of the structure the procedure uses a modal combination rule with previously scaled modal responses according to some “inter-modal scale factors”. In the practical version of the method these factors are simplified as constant each time a sequential modal spectrum analysis is carried out. In the method, once the modal capacity curves are defined, the modal performance of the structure is obtained by using, for each mode, the equal displacement rule with consideration of the short period correction. To obtain the global performance of the structure an accepted modal combination rule is
used. This method is different to pushover-based procedures as equivalent static forces are never applied to construct the modal capacity diagrams. Instead, the method uses displacements derived from consecutive modal analysis to obtain the different segments of the modal capacity diagrams corresponding to different performance stages.

### 1.1.3 Method based on the Substitute Structure

One of the widely used methods for the displacement based evaluation/design of bridges is one in which the original structure is substituted by a linear viscoelastic counterpart, e.g. Kowalsky [2002]. This “substitute” structure has the same configuration as the original, with equivalent stiffness and damping properties assigned to the elements where damage actually occurs or it is assumed to occur, when performing evaluation and design applications, respectively.

The concept of introducing viscous damping to represent energy dissipation characteristics of a system was first presented by Jacobsen [1960]. However, the first known earthquake engineering application of this idea to approximately substitute a hysteretic SDOF system subjected to earthquake action by a viscoelastic one was investigated by Rosenblueth and Herrera [1964].

For the assessment of real structures, Gulkan and Sozen [1974] introduced formally the concept of substitute structure for a SDOF structure comparing the resulting analytical results with the corresponding experimental ones. Later on, Shibata and Sozen [1976] extended this formulation to MDOF systems by proposing an approximation to define the modal damping ratio of the whole structure as a weighted average of the element damping ratios. In this approximation, once the equivalent linear stiffness of the elements and the modal damping ratio of the structure are determined, modal spectral analysis may be used to approximately evaluate its seismic performance.

Recent papers by Blandon and Priestley [2005] and Dwairi [2004], Guyader and Iwan [2006], among others, present a thorough list of different definitions of equivalent viscous damping, $\xi_{eq}$, and where applicable, effective periods, $T_{eff}$. In this section all proposed definitions are not presented, referring the reader to Ayala et al. [2007].

#### 1.1.3.1 The substitute structure applied to the displacement based evaluation of bridges

In this section it is assumed that for evaluation purposes the bridge structure under consideration is already designed and that the substitute structure method is used to assess its seismic performance when subjected to a seismic demand given by a design spectrum. The steps involved are illustrated in Figure 1.1 and are:
Guidelines for Displacement-Based Design of Buildings and Bridges

Figure 1.1: Evaluation procedure for the method based on the substitute structure

Step-1: Determination of the inelastic behaviour of the pier sections, as moment vs. curvature, within the potential damaged region.

Step-2: Determination of the load-displacement characteristics at the top of the piers. Based on the moment vs. curvature curves determined in step 1 and on an assumption for the length of the plastic hinge, load-displacement curves for the top of the piers are constructed, considering the different maximum lateral displacement (ductility) levels.

Step-3: Determination of equivalent linear viscoelastic properties of the piers. Based on the nonlinear force vs. displacement curves of the piers determined in step 2, the equivalent linear viscoelastic properties of the piers, e.g., secant stiffness, $K_{eff}$, and equivalent viscous damping ratio, $\xi_{eq}$, at maximum displacement, are calculated. A procedure to find the properties determined in steps 1 through 3 is proposed and exemplified in Section 2.2.2 of the present Part II.

Step-4: Construction of the curves for each pier depicting the variation of the equivalent stiffness and damping ratio in terms of displacement ductility. To consider the transient nature of the earthquake action in the equivalent properties curves, it is necessary to include a modification factor that takes into account the fact that the maximum displacement attained during an earthquake occurs only a very limited number of times, e.g., for ordinary narrow band records, equivalent properties associated to the maximum displacement multiplied by a factor equal to 0.67 have shown to be a good approximation.
Step-5: Initiation of the iterative procedure for performance determination. Since the equivalent viscoelastic properties of the piers are functions of the associated maximum displacements, it is required to initially assume a distribution of maximum displacements under design conditions. A simple way to obtain this distribution of maximum displacements is to carry out a modal spectral analysis considering for the piers the initial stiffness and the inherent modal viscous damping for this type of structures.

Step-6: Assumption of the performance of the bridge. The distribution of maximum displacements is made equal, for the first iteration, to the displacements obtained from step 5, and for subsequent iterations, to the displacements obtained from step 8.

Step-7: Determination of the performance of the bridge. Assuming an elastic bridge deck, modal analysis is carried out on the bridge structure, with viscoelastic properties of the piers determined from the equivalent properties curves derived in step 4 at the maximum displacement distribution assumed in step 6. The equivalent modal viscous damping ratios for the bridge can be evaluated for each mode as the sum of the inherent modal damping, $\xi_o$, and that corresponding to the weighted average of the equivalent hysteretic damping ratios for all the structural elements using the original approach of Shibata and Sozen [1976].

Step-8: Update the performance of the bridge. The distribution of maximum displacements is made equal to the distribution of displacements computed from step 7.

Step-9: Comparison of the updated and the assumed performances. When, during the iteration process, the ratio between the updated (from step 8) and the assumed (from step 6) performances is within an accepted tolerance, the process is stopped and the performance of the bridge equals the displacement distribution from step 8, otherwise steps 6 through 9 are repeated.

1.1.3.2 The substitute structure applied to the displacement based design of bridges

A similar procedure to that described above for evaluation may be used for the DDBD of bridge structures. The design procedure proposed in this chapter is derived following similar steps to those presented in the above section and it is different to that presented by Kowalsky [2002] inasmuch as it includes information about a target damaged distribution under design conditions, includes the participation of all contributing modes and uses as basic design information the relations between the inelastic deformation at the top of the piers vs. local curvature demands at the hinges at the base of the damaged piers.
To apply this procedure it is necessary to have, for different pier geometries and acceptable design configurations, design curves similar to those illustrated in Section 2.2.2 of the present Part II. The procedure is schematically shown in Figure 1.2 and the steps describing its application are described in the following:

**Step-1**: Perform a conventional force design for permanent plus vehicular plus earthquake loads, choosing an acceptable target design index, e.g., a global ductility factor.

**Step-2**: Considering an elastic bridge deck and based on the results of step 1, check if the damage distribution, obtained by comparing the maximum displacements at the top of the piers with their corresponding yield displacements, is acceptable, in which case go to step 3, otherwise modify the design of those piers where no damage is accepted to occur or where the ductility demand is not acceptable, and go to step 1.

**Step-3**: Calculate the additional damping ratios and reduced stiffness for the damaged piers as presented in steps 1 through 4 of Section 1.1.3.1 above, and perform the seismic analysis of the corresponding substitute structure.
Step-4: Compare the calculated maximum pier displacements with those considered as target in the design. Based on this comparison and on the information presented in Section 2.2.2 of the present Part II, modify, if required, the design of the piers and calculate the new local ductility demands of the damaged piers and go back to step 3, otherwise go to step 5.

Step-5: The design of the bridge has converged to an acceptable target performance distribution. It is important to mention that if the target performance is not reached, this could be due to the choice of an unfeasible damage distribution, in this case an alternative distribution should be considered.

1.1.4 Method based on the Non-linear Capacity of the Structure

From the detailed study of the existing procedures for evaluation and design of bridges based on the non-linear capacity of structures, it has been found that, in general, all involve the following two tasks:

1. Determination of the deformation capacity of the structure and its corresponding strength for the sequential formation of the events (e.g., plastic hinges) associated to predefined limits states and the corresponding redistribution of the seismic forces which act on the structure.

2. Determination of the seismic performance using displacement/acceleration design spectra; considering SDOF systems (one or several systems, depending on the method) whose non-linear force-displacement relationships are derived from the results of step 1. The use of smooth spectrum produces, for evaluation purposes, the maximum displacement, i.e., the displacement demand for a given design, and for design purposes, the strength demand for a required displacement.

Based on the same concepts which support these two tasks, a performance evaluation/design method is proposed using the same hypotheses and considerations as the method developed by Ayala [2001], which explicitly considers the non-linear behaviour of the structure on the derivation/postulation of a target response curve of a reference SDOF system considering the participation of all modes to determine the performance of the otherwise MDOF of the structure under evaluation/design. The characteristics of the response curve of the reference SDOF system are obtained from the calculated/desired distributions of damage for the considered design objective. In this method, the design seismic demands associated to each of the design objectives are concurrently determined using the characteristics of the calculated/assumed response curve of the reference SDOF.
The evaluation version of this method is an evolution of the procedure proposed by Requena and Ayala [2000]. In this method the maximum displacement of the reference system is obtained from one of the different variations of the equal displacement rule, e.g., Fajfar [1999] and Ruiz-Garcia and Miranda [2004], and directly transformed to the maximum displacement of the structure by an ad hoc modal spectral analysis. This method is similar to the IRSA method, however, it differs in the way in which higher modes are considered. The details of the application of this method are presented in Ayala et al. [2007].

A key question in the application of displacement-based evaluation/design methods to MDOF structures is how to transform global performance into demands of local inelastic deformation in the individual structural members. In this respect, detailed procedures intended to achieve this purpose are, for example, those proposed by Seneviratna and Krawinkler [1997], however a definite solution to this problem has not been established and it is still the topic of current investigations.

1.1.4.1 Non-linear capacity concept applied to the displacement based evaluation of bridges

The application of the proposed method involves the following steps, schematically illustrated in Figure 1.3:

**Step 1.** The seismic demand is defined by a smooth response spectrum corresponding to a chosen seismic demand level.

**Step 2.** The response curve of the reference SDOF system is obtained through a series of Modal Spectral Analyses (MSA) as outlined in Ayala et al. [2007], considering as many damage stages developed by the structure as necessary, until its maximum capacity is reached. The contribution of higher modes of vibration in the response curve is taken into account using a modal combination rule (e.g., SRSS or CQC). In this work, a damage stage is defined every time a plastic hinge is formed at the end section of a pier.

**Step 3.** For each damage state $j$, the corresponding MSA results are used to calculate the scale factor, $Sf(j)$, at the base of each damaged pier using the equations presented in Ayala et al. [2007]. The lowest scale factor corresponds to the pier requiring the lowest seismic demand to yield.

**Step 4.** The scaled pseudo-acceleration, $\Delta \tilde{a}$, and the scaled spectral displacement, $\Delta \tilde{d}$, corresponding to the period of the dominant mode of the structure in the $j^{th}$ damage stage, are defined from the scaled spectrum, using an acceleration vs. displacement format, ADRS, which is the same format in which the response curve is defined.
Step 5. The capacity of the structure is reached when a local or global instability occurs, indicating that the construction of the response curve is finished and that the methodology for the evaluation of the spectral displacement may be continued. Otherwise, a new damage stage has to be considered and a new MSA performed for the determination of the next point on the response curve.

Step 6. The inelastic displacement demand, or performance spectral displacement, $S_d^*$, may be calculated via the equal displacement rule [Veletsos and Newmark, 1966], with the short period correction [Fajfar, 1999], as specified in Annex B of EC 8 [CEN, 2004] or by considering more recent results [e.g., Ruiz-Garcia and Miranda 2004].

Step 7. When the available capacity of the structure exceeds the demand, a new scale factor, $N_s$, needs to be calculated for the first point ("point $j$") of the response curve where the displacement is larger than the performance displacement. This is done in accordance with the equations presented in Ayala et al. [2007].

The seismic performance of the bridge for the selected performance parameter, in this case the maximum lateral pier displacement, is finally calculated as the weighted sum of the corresponding parameters of the $N$ modal spectral analyses performed until the target performance displacement is reached.
1.1.4.2 **Non-linear capacity concept applied to the displacement based design of bridges**

The overall process to design a bridge to meet a performance level defined by a target

design ductility is schematically shown in Figure 1.4; it consists of the following steps:

**Step 1.** The response curve of a reference system corresponding to the mode of the
structure with the highest contribution is constructed by considering two structures with
different dynamic properties: one with properties derived from the bridge without
damage corresponding to a pre-designed structure; the other, the same bridge with
reduced properties to incorporate a proposed damage distribution expected to occur
under design demands.

**Step 2.** The strengths of the bridge piers where damage is accepted to occur are
determined from a MSA using the dynamic properties of the undamaged bridge and the
elastic design spectrum reduced by a factor defined from the strength spectrum for a
system with a global performance index estimated from a design pier displacement. The

![Figure 1.4: Design procedure for the method based on the reference structure](image-url)
complementary strengths for the bridge elements where damage is not admitted, corresponding to the second stage, are obtained from a second MSA using the properties of the damaged bridge and the same elastic design spectrum scaled to consider a seismic demand that added to that considered for the first stage gives the total seismic demand.

Step 3. The final design forces are obtained by summing the element forces of the two previously defined MSAs and combining them with the element forces from the analysis for gravity and traffic loads in accordance with the bridge design code.

1.1.5 Application Examples

To illustrate the application and validate the accuracy and potentiality of the proposed methods, six sample bridge structures are evaluated. The first structure is a scaled four span single supported concrete bridge tested at ELSA with a variety of pseudo-static and pseudo-dynamic tests [Pinto et al., 1996], while the other five structures have the same configuration as the first, but with different dimensions and characteristics of the piers and superstructure, as designed by Isaković and Fischinger [2006]. The bridges are all reinforced concrete structures designed in accordance with current seismic codes. The general layout of the considered bridges and the geometric and structural characteristics of each bridge are described in Ayala et al. [2007]. For all bridges the seismic design level was defined using different intensities of the EC8 design spectrum [CEN, 2003] corresponding to soil type B, 5% damping ratio and a 1.2 soil amplification factor.

The equivalent properties derived for rectangular reinforced concrete hollow piers described in Section 2.2.2 of the present Part II were used to construct the substitute structure of the sample bridges analysed. The seismic demands were represented by artificial records compatible with the EC8 design spectrum, with peak accelerations of 0.35g and 0.70g for the ELSA bridge, and peak accelerations ranging from 0.20g to 0.70g for the other five bridges. The calculated performances for all bridge examples considered are described in Ayala et al. [2007].

1.1.6 Concluding remarks

This chapter presents two different methods of displacement based evaluation/design of bridges which improve previously developed approximations. The results presented may be directly used to construct a “substitute” structure or a response curve of a reference SDOF system which lead to a sought performance or to a design for a specified design objective defined by a maximum pier displacement and earthquake intensity.

Both proposed methods may be considered enhanced versions of others currently under use or investigation by other research groups, as they take into consideration the
contribution of higher modes of vibration and the displacement reversal nature of earthquake action through evolving modal spectral analyses, rather than from evolving force or displacement based pushover analyses.

The work presented shows that the evaluation and the design options of the proposed methods, may give acceptable results with limited computational effort as long as the structure is “regular”. This conclusion unfortunately may not be extended to the case of “irregular” bridges, where the relative relevance to performance of highly correlated modes changes as a function of the earthquake intensity.

It is shown that the use of these methods with response spectra as seismic demand may not guarantee correct results for all seismic design levels consistent with the spectrum; e.g., the results presented in Ayala et al. [2007] for the V213P bridge are not satisfactory compared with those of the statistical study of 1000 non-linear time history analyses. For this example, the observed lack of approximation may be due to the fact that for the considered design level, the bridge, due to the occurrence of new damage, changes its instantaneous fundamental mode shape from rotational to translational, thus becoming irregular. It is evident that more research is needed to fully understand why this lack of approximation occurs, to determine for which combinations of bridge configurations and seismic design levels the application of the proposed methods is reliable.

Preliminary results show that for bridges with a significant contribution of higher modes and with large non-linearities, the methods proposed, in particular the one based on the non-linear capacity of the structure, lead to better results than alternative simplified procedures based on a “substitute structure” and on an “equivalent” SDOF system, which do not explicitly consider the contribution of higher modes. For the design versions of the methods proposed, the deformation capacity of the structure is obtained from an assumed damage distribution, explicitly defined in the design process.

It is shown that the methods presented may be carried out with commercial analysis software. In particular, for the method based on the non-linear capacity of the structure, the response curve of the reference system is constructed using partial results of evolving modal spectral analyses. This approach is simpler and superior to others currently used, as it does not depend on results of non-linear pushover analyses with evolving lateral force or displacement patterns. However, it is accepted that the application of modal spectral analysis with accepted modal combination rules for the evaluation/design of bridges gives results consistent with calculated/expected maximum performances.
1.2 DESIGN OF BRIDGE PIERS DIRECTLY ON THE BASIS OF DISPLACEMENT AND DEFORMATION DEMANDS, WITHOUT ITERATIONS WITH ANALYSIS

1.2.1 Proposed displacement-based design procedure

A procedure is proposed for the direct design of piers on the basis of displacements and deformations, without an unduly large number of iterations between analysis and member verifications. The procedure focuses on concrete piers monolithically connected to a prestressed concrete continuous bridge deck. It comprises the following steps:

**Step-1:** Dimensioning of the deck and of the piers for:

- the Ultimate Limit State (ULS) under the combination of factored permanent and transient actions, at the all relevant intermediate stages of construction as well as in the completed bridge configuration, taking into account the redistribution of action effects due to creep and losses of prestress, etc., as appropriate;

- the Serviceability Limit State (SLS), for the relevant combination of permanent and transient actions in the completed bridge configuration, as relevant.

**Step-2:** Establishment of the effective stiffness of the piers and of the deck, \((EI)_{\text{eff}}\), which is representative of the elastic stiffness of the corresponding members during the seismic response (longitudinal or transverse):

- Regarding the deck, if the action effects of the design seismic action are considered to be relatively low, so that cracking is not expected to take place under the design seismic action (acting together with the concurrent quasi-permanent gravity loads and the prestress), then the elastic stiffness, \((EI)_{c}\), of the uncracked gross concrete section should be taken as effective stiffness. This is normally the case for prestressed decks. Where, by contrast, cracking is expected to take place under this combination of the design seismic action and gravity loads, etc. the cracked stiffness of the deck section should be used, evaluated from a moment-curvature diagram of the section. In asymmetric sections, such as box sections about their horizontal centroidal axis, the mean value of the cracked stiffness for positive or negative bending should be used.

- Regarding the piers, the value of \((EI)_{\text{eff}}\) should normally be the secant stiffness to yielding of the end sections where plastic hinges are expected to develop under the combination of the design seismic action with the relevant gravity loads: at the base of each pier and at its connection to the deck for the longitudinal seismic action, or at the base of each pier alone for the transverse seismic action. The secant stiffness to yielding of the plastic hinge sections, \((EI)_{\text{eff}}\), depends on the geometry of the pier
section and on its axial load, but also on the moment-to-shear ratio ("shear span") at these sections, as well as on the amount and arrangement of longitudinal reinforcement. In Step 2, \( (EI)_{\text{eff}} \), can be conveniently estimated through the empirical expression given in Section 2.2.1.4 of Part II, using parameters that are known before dimensioning the piers for the seismic action, such as the geometry of the pier section, its axial load and the shear span at the locations of expected plastic hinges.

**Step-3**: Estimation of chord rotation demands, elastic or inelastic, at the pier ends, due to both horizontal components of the full (unreduced) design seismic action, through modal response spectrum elastic analysis for 5%-damping.

**Step-4**: Selection of a target value of chord rotation ductility factor, \( \mu_\theta \), at the plastic hinges expected to form at pier ends under the design seismic action and estimation of the corresponding chord rotation at yielding there as:

\[
\theta_\text{y} = \theta_\text{Ed} / \mu_\theta
\]

where:

\( \theta_\text{Ed} \): chord rotation demand at the end section of the pier from Step 3.

This gives the opportunity to target a uniform level of inelasticity between piers of different height, etc. Then, the pier yield moments, \( M_y \), corresponding to the chord rotations at yielding from Eq. (1.1) are determined as:

\[
M_y = 3EI_{\text{eff}} \theta_\text{y} / L_S
\]

on the basis of the same values of the secant stiffness to yielding at the pier end sections, \( (EI)_{\text{eff}} \), and of the shear span, \( L_S \), used in Step 2. The vertical reinforcement of the pier is then dimensioned to provide the values of yield moments, \( M_y \), from Eq. (1.2).

**Step-5**: Verification of the piers for the inelastic deformation demands estimated in Step 3 for both horizontal components of the design seismic action. Piers are verified against the so-estimated deformation demands on the basis of the condition:

\[
\theta_\text{Ed} \leq \theta_\text{Rd}
\]

where:

\( \theta_\text{Ed} \): chord rotation demand at the end section of the pier from Step 3 (cf. Eq. (1.1)); and

\( \theta_\text{Rd} = \theta_{\text{UK},0.05} / \gamma_{\text{Rd}} \): design value of pier chord rotation capacity, determined from a 5%-fractile, \( \theta_{\text{UK},0.05} \) of the ultimate chord rotation of the pier, divided by appropriate
resistance partial factor, $\gamma_{Rd}$. The models in Section 3.1.1.1 of the present Part II are used for the calculation of the expected value of the pier ultimate chord rotation, $\theta_{um}$ and of $\theta_{uk,0.05}=0.5\theta_{um}$ from it. If the design seismic action has a mean return period of 475 years (i.e., the one normally used for the “Life Safety” performance level of ordinary new buildings), an appropriate value of $\gamma_{Rd}$ is $\gamma_{Rd}=2$.

The transverse reinforcement of the pier is the main free dimensioning variable controlling the value of $\theta_{um}$ and should be dimensioned to fulfill Eq. (1.3).

**Step-6:** Verification of the piers in shear and dimensioning of their transverse reinforcement, so that:

- pre-emptive, brittle shear failure before flexural plastic hinging does not develop; and
- piers can safely sustain the cyclic degradation of shear resistance in the plastic hinge due to the inelastic deformations predicted for the plastic hinge from the analysis in Step 3.

Pre-emptive brittle shear failure is prevented by dimensioning the entire height of the pier to fulfill the condition:

$$V_{CD} \leq V_{Rd,mon}$$  \hspace{1cm} (1.4)

where:

- $V_{CD}$: “capacity design” shear force, determined on the basis of equilibrium, assuming that piers develop their overstrength moment capacity at both top and bottom sections if bending is within a vertical plane in the longitudinal direction of the bridge, or only at the base, for bending within a vertical plane in the transverse direction; the overstrength moment capacity of the pier is taken as the design value of its moment capacity, $M_{Rd}$, times an overstrength factor, $\gamma_o$. In the present case, a value $\gamma_o=1.55$ was found necessary to cover the effect of steel strain hardening and the difference between the mean and the design values of material strengths.

- $V_{Rd,mon}$: design value of shear resistance, for monotonic loading by non-seismic actions.

Verification of plastic hinges in piers against (ductile) shear failure after pier flexural yielding is of the following form:

$$V_{CD} \leq V_{Rd,cyc} (\mu)$$  \hspace{1cm} (1.5)

where:

- $V_{CD}$: “capacity design” shear, as in Eq. (1.4); and
$V_{Rd,cyc}(\mu_0)$: design value of shear resistance in cyclic loading after flexural yielding, calculated using the design values of material strengths, according to Section 3.1.1.2 of the present Part II, as a decreasing function of the chord rotation ductility factor, $\mu_0$. The value of $\mu_0$ is taken equal to the chord rotation demand at the end section of the pier from the analysis in Step 3 (i.e. to $\theta_{Ed}$ in Eq. (1.1)), divided by the corresponding chord rotation at yielding of this end section, $\theta_y$, calculated from the expressions in Section 2.2.1.3 of Part II, as a function of the amount and arrangement of the pier vertical reinforcement already dimensioned in Step 4.

The value of $V_{Rd,mon}$ to be used in Eq. (1.4) outside the plastic hinge regions may be taken equal as $V_{Rd,mon}=V_{Rd,cyc}(\mu_0=1)$, with $V_{Rd,cyc}(\mu_0)$ according to Section 3.1.1.2 of Part II. The transverse reinforcement and the thickness of the pier are the main dimensioning parameters controlling the values of $V_{Rd,mon}$ and $V_{Rd,cyc}(\mu_0)$, to fulfill Eqs. (1.4), (1.5).

**Step-7:** The secant stiffness to yielding at the pier end sections, $(EI)_{eff}$, is updated using the expressions in Sections 2.2.1.1-2.2.1.3 of Part II, on the basis of the amount and arrangement of the pier vertical reinforcement from Step 4 and of the shear span (moment-to-shear ratio), $L_s$, from the elastic analysis results in Step 3. If the so-computed values of $(EI)_{eff}$ deviate significantly from the values of $(EI)_{eff}$ determined for the piers in Step 2 and used in Step 3 for the estimation of the deformation demands, then Steps 3 (i.e., the elastic analysis) to 6 (the verifications) are repeated, until full consistency with the effective stiffness used in the analysis is achieved.

### 1.2.2 Nonlinear modelling of bridges with monolithic deck-pier connection

A major part of the work focused on the development of a simple procedure for the estimation of pier inelastic deformation demands (chord rotations), through elastic modal response spectrum analysis of the type suggested for step 3 of the proposed displacement-based design procedure, extending therefore the applicability of the “equal displacement” rule to the level of member deformations. If this rule is applicable, member deformations may be estimated without recourse to pushover analysis to establish the correspondence between the global displacement demand and local deformation demands (plastic hinge or chord rotations. This has been achieved through comparisons of results of nonlinear dynamic (time-history) and modal response spectrum analysis for a representative set of bridges. To this end a computational capability has been developed for modelling, seismic response analysis and evaluation of concrete bridges. The computational tool is program ANSRuop-Bridges, developed at the University of Patras, Structures Laboratory, as a significantly improved and expanded version of the ANSR-I program developed at UC Berkeley [Mondkar and Powel 1975]. The analysis capabilities of ANSRuop-Bridges comprise:
1. Determination of the initial condition for the seismic response analysis of the bridge, by linear static analysis for permanent loads and traffic loads that act concurrently with the seismic action. Individual deck tendons are replaced by statically equivalent loads:

- distributed loads in 3D, equal to the prestressing force after losses times the 2nd derivative of the eccentricities of the tendon from the longitudinal axis of the deck,

- concentrated forces and moments at tendon deviations or anchorage, equal to the unbalanced components of prestressing force after losses at the point of deviation or anchorage and their moments with respect to the deck longitudinal axis, etc.

2. Eigenmode/eigenvalue calculation and response spectrum elastic analysis for user-specified or default elastic response spectra in the transverse or longitudinal direction, with Complete Quadratic Combination of peak modal contributions.

3. Linear static seismic response analysis, under transverse or longitudinal lateral forces proportional to nodal masses and a specified response acceleration pattern. Response accelerations considered are constant over the deck and inverted triangular up the piers, or consistent with the normal mode having the largest participating mass in the horizontal direction considered. The 5%-damped response spectrum is entered at the fundamental period in that direction, estimated via the Rayleigh quotient, to determine the magnitude of lateral forces.

4. Nonlinear static ("pushover") seismic response analysis, under increasing transverse or longitudinal lateral forces proportional to either of the force patterns in 3 above.

5. Nonlinear dynamic analysis, under one or two earthquake components (time-histories).

ANSRuop-Bridges was developed from ANSR-I using an object-oriented approach. A full-feature pre-processor allows user-friendly graphical definition of the geometry, the restraints and external loads and placement of tendons and longitudinal and transverse non-prestressed reinforcement in the section. Circular or rectangular piers (solid or hollow) are provided. Options for the deck include T-girders and single-cell box sections with vertical or inclined webs, tapered outhangs and internal haunches in the box.

On the basis of the nonlinear $\sigma$-$\varepsilon$ laws of steel and concrete, the pre-processor constructs moment-curvature diagrammes of any section at the pier or the deck for the initial value of the axial force due to the loads acting concurrently with the seismic action and the pre-strains of the tendons. The points of decompression, cracking, yielding or flexural failure of the section are automatically identified in these diagrammes and the secant stiffness to
these points is determined. The pre-processor computes the shear and torsional rigidity of the section, including or not the effect of cracking (the default option being user-specified fractions of the elastic torsional or shear rigidity of the uncracked gross section).

A full-feature post-processor [Kosmopoulos and Fardis 2006] produces fully graphical displays of plots of forces, displacements, deformations or damage (demand-capacity ratio in flexure or in shear) from the analysis. For nonlinear dynamic analysis under a suite of input ground motions, the maximum, minimum, mean value and coefficient-of-variation of response measures for the full suite of input motions is displayed. The post-processor allows verification of the deck and the piers on the basis of the most adverse response results. Flexure-governed behaviour is verified in terms of chord-rotations at pier ends and of deck curvatures, on the basis of the expressions in Section 3.1.1.1 of the present Part II. Diagonal tension failure before or after flexural yielding, web crushing or diagonal compression failure are verified according to Section 3.1.1.2 of Part II on the basis of shear forces.

For the piers, the expressions in Section 2.2.1.4 of Part II are used to compute the yield moment and the effective stiffness at yielding of the pier end section(s). For nonlinear analysis, the values of these properties, as well as of the shear resistance and its reduction with increasing cyclic inelastic deformations and of the ultimate chord-rotation at the pier ends, are updated during the response, whenever the value of any parameter influencing these properties (the axial force, \( N \), the shear span \( L_s \), the location of the neutral axis in the section, the peak inelastic deformation demand to the current point of the response, etc.) changes beyond a certain tolerance.

The deck and the piers are discretized longitudinally into a series of prismatic beam elements in 3D, with inelasticity lumped at point hinges at the element ends. The moment-rotation relation of the point hinges of each deck element, derived from the moment-curvature relationship of the deck section, is multilinear, with corners at cracking and yielding of the section. The tensile strength of concrete is normally neglected, meaning that cracking coincides with decompression. The effective stiffness of all the elements into which each pier is discretized, \((EI)_{\text{eff}}\), is computed according to Section 2.2.1.4 of Part II, considering the pier as a whole: the pier shear span \( L_s \) is the moment-to-shear-ratio at the yielding end the pier (for deck monolithically connected to the piers, \( L_s \) is generally half the pier clear height, or the full clear height of single piers for the transverse component of the seismic action). Inelasticity is taken independent and uncoupled in the two transverse directions of the pier, but M-N interaction is considered in each direction (\( M_y-N \) and \( M_z-N \)), as well as the effect of the variation of axial force during the response. P-\( \Delta \) (2nd-order) effects are included. Monolithic joints between the deck and the piers are considered as rigid, but slippage of the vertical bars of the pier from the joint is accounted for, by including the effect of the resulting fixed-end rotation.
of the pier top sections within the corresponding secant-to-yielding stiffness of that end of the pier. Fixed-end rotation is similarly included at the base of the pier, where it is considered fixed at its foundation.

1.2.3 Nonlinear dynamic vs modal response spectrum analysis

Several bridges were subjected to nonlinear dynamic (time-history) and modal response spectrum analyses, using the same initial stiffness and the same 5%-damped elastic spectrum (not reduced by \( q \)) to which the seven input motions conform. Results are compared here for a representative bridge (see Figure 1.5). The bridge has a constant deck section, with a depth of 2.5m and a width of 11.3m at the top slab and 5.5m at the bottom slab; piers are circular, with a diameter of 1.2m. At the abutments the deck is free to slide longitudinally but fully restrained transversely. It was designed for the Type 1 spectrum on type C soil in Eurocode 8 for a peak ground acceleration (PGA) of 0.14g on rock (0.16g on soil C). The analyses use the mean values of material properties, and is performed for seismic action separately in the longitudinal and transverse direction with PGA on rock 0.25g, 0.35g and 0.45g. The nonlinear time-history analyses use seven input motions emulating the strongest of the two horizontal components of seven historic earthquakes, modified to fit the 5%-damped Eurocode 8 Type I spectrum on type C soil.

![Graphs showing results of nonlinear dynamic and modal response spectrum analyses.]

Figure 1.5: (a) Geometry of bridge T6-2; (b) pier section and moment-curvature diagrammes; (c) mid-span deck section and moment-curvature diagrammes in each bending direction (Yellow circle: Yielding of reinforcement in piers or in the deck for negative moment \( M_{yy} \), decompression of deck in all other cases. Red circle: rupture of reinforcement in pier, or in deck for negative moment \( M_{yy} \), yielding of reinforcement in all other cases for the deck).

Figure 1.6 shows the average over the seven nonlinear dynamic analyses of the maximum deformation demand (chord rotation) during the response, divided by the corresponding elastic value from the modal response spectrum analyses. Figure 1.7 illustrates the
magnitude of the nonlinearity induced by the input motions, as the average over the seven nonlinear dynamic analyses of the maximum deformation demand during the response, divided by the deformation at deviation from linear moment-curvature behaviour (yellow circle in the example diagrammes of Figure 1.5, signifying yielding of the reinforcement in the piers or cracking in the deck - with the concrete tensile strength neglected here - except for the side of the section opposite to the mean tendon, where it signifies yielding of the reinforcement). Figure 1.8 shows the average over the seven nonlinear dynamic analyses of the maximum deformation demand during the response, divided by the deformation at yielding of deck reinforcement or rupture of reinforcement (red circle in the example moment-curvature diagrammes of Figure 1.5).

Figure 1.6: Ratio of mean inelastic chord rotation demand from nonlinear dynamic analyses to value from modal response spectrum analysis.
Comparisons were also made between the predictions of nonlinear dynamic and nonlinear static (pushover) analysis. The overall conclusions are the following:

- Regarding the piers: At a PGA level of 0.25g, both nonlinear static and 5%-damped modal response spectrum analysis give, on average, satisfactory predictions of inelastic chord rotation demands at pier plastic hinges, with modal response spectrum analysis predictions being slightly better. For stronger ground motions the elastic response spectral or the nonlinear static analysis predictions of deformations become greater (and therefore more conservative) than those from nonlinear dynamic analysis.
- Regarding the deck: For seismic action in the transverse direction, nonlinear static analysis slightly underestimates deck deformations. However, this is of little practical relevance, as no significant yielding of reinforcement or tendons occurs in the deck even under very strong ground shaking. This underestimation decreases for stronger ground motions. By contrast, if modal response spectrum analysis is used, the stronger the ground motion, the more conservative are the elastic predictions of deformations in both directions. When all piers form plastic hinges, deformations predicted by modal response spectrum analysis at deck sections where cracking (i.e., decompression) or yielding of the reinforcement takes place are close to those from
nonlinear dynamic analysis. Otherwise, the deformations from modal response spectrum analysis exceed those from nonlinear dynamic analysis by a factor of 1.2, on average.

Figure 1.9: Mean peak shear force demand from nonlinear dynamic analyses divided by shear resistance. Bridge of conventional force-based design.

Figure 1.9 shows the average over the seven nonlinear dynamic analyses of the peak shear force deformation demand during the response, divided by the corresponding shear resistance, computed according to Section 3.1.1.2 of Part II using mean values of material strengths. The values in this figure and in Figure 1.8, interpreted as damage ratios in shear and in bending, respectively, suggest that the conventional, force-based design of this...
bridge provides a very large safety margin, even under ground motions much stronger than its design seismic action.

1.2.4 Application of the proposed design procedure and evaluation of the design

The proposed procedure was applied for the displacement-based design of several bridges. The application to the bridge of Figure 1.5 is exemplified here. The resulting
design is evaluated and compared to that of the original bridge, in terms of the amount of pier reinforcement and of the bridge response and performance under a seismic action in the longitudinal or transverse direction, with PGA on rock of 0.25g, 0.35g and 0.45g. In this respect, Figures 1.10-1.12 are the counterparts of Figures 1.7-1.9.

Figure 1.11: Mean peak deformation demand from nonlinear dynamic analyses divided by value at yielding of tendons or fracture of reinforcement. Bridge with proposed displacement-based design.

In the new design piers have vertical reinforcement ratios of 0.21%, instead of 1.006% originally. They also have volumetric transverse steel ratio of 0.269% or 0.236% in the plastic hinge regions at the base or at the top, respectively, and 0.135% between them, vs 0.424% at both plastic hinge regions and 0.212% in-between, in the original design. As a result, pier ductility demands are larger in Figure 1.10 than in Figure 1.7, but flexural damage ratios are lower in Figure 1.11 than in Figure 1.8. So the proposed displacement-based design gave better performance and savings in reinforcement. In addition, although
the deck is the same as before, it is subjected to much less inelastic action than originally. Obviously the new piers have a large safety margin in flexure or shear, even for seismic actions more than three-times the design one. Note that, owing to the very low vertical reinforcement ratio of the new piers, the reinforcement-independent empirical expression for $(EI)_{eff}$ in Section 2.2.1.4 of Part II, fitted to test data on piers with much higher steel ratio, significantly overestimates the pier secant stiffness to yielding. So, a repetition of Steps 3-6 of the procedure was triggered by updating the value of $(EI)_{eff}$ in Step 7 using the expressions in Sections 2.2.1.1-2.2.1.3 and the actual pier reinforcement. The final new design is almost twice more flexible than originally perceived in Steps 2 and 3, as evidenced by a difference of almost 40% in the fundamental periods.

Figure 1.12: Mean peak shear force demand from nonlinear dynamic analyses divided by shear resistance. Bridge with proposed displacement-based design.
2 ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BRIDGES

2.1 ANALYSIS METHODS FOR ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMANDS IN BRIDGES

2.1.1 Procedures for estimation of pier inelastic deformation demands via nonlinear analysis (static or dynamic) of the bridge

2.1.1.1 Introduction

During the past years there has been increasing interest in the research community for the formulation of practical design procedures based on the concepts of displacement-based design. The main problems that need to be addressed are the estimation of the inelastic deformation capacity of the ductile elements of the structure and the evaluation of the seismic deformation demands. In this section these two fundamental problems of displacement-based design are investigated within the framework of typical bridge applications. An analytical procedure is presented for the determination of the deformation capacity of typical bridge piers; the results are verified with published test data. A set of typical bridge configurations is then selected and designed on the basis of conventional spectral analysis employing the $q$-factor approach. The nonlinear response of the selected bridges is evaluated via extensive nonlinear analyses (static, pushover and non-linear dynamic). The comparison of the analyses results is used for the development of simple proposals for the estimation of pier inelastic deformation demands.

2.1.1.2 Deformation capacity of reinforced concrete piers

The deformation capacity of reinforced concrete (RC) piers is very important for the determination of their response and the response of the whole structure under earthquake loading. The simplest procedure for the calculation of the effective stiffness and the available deformation capacity of RC members includes an assumption for the bending behaviour by a moment-curvature relation, which is integrated along the member length. The actual curvature distribution can be modelled in a simplified manner by elastic and inelastic regions. The contribution of the elastic region can be considered by the well known equation of beam theory:
\[ \varphi = \frac{M}{EI} \]  

(2.1)

where \( EI \) is the elastic stiffness before yielding.

The contribution of the plastic region can be considered by making the assumption that the inelastic deformation is caused by the total plastic rotation \( \theta_p \) which is developed in the plastic hinge region. This approximation is also valid at failure, where \( \varphi_{\text{max}} = \varphi_0 \). Thus the plastic rotation at failure is equal to:

\[ \theta_{p,u} = (\varphi_u - \varphi_y) L_p \]  

(2.2)

For a simple cantilever member the chord rotation at failure is given as:

\[ \theta_u = \varphi_y \frac{L_y}{3} + (\varphi_u - \varphi_y) L_p \left( 1 - 0.5 \frac{L_{pl}}{L_s} \right) = \theta_y + (\varphi_u - \varphi_y) L_p \left( 1 - 0.5 \frac{L_{pl}}{L_s} \right) \]  

(2.3)

For the case of bending-shear failure (appearance of flexural and shear cracks) the relations above provide approximate results, because the moment-curvature diagram corresponds approximately to the internal moment-curvature diagram of the section.

In the past there have been proposed advanced models incorporating the contribution of the effects of the shear cracks, the possible reinforcement slippage, buckling of the reinforcement bars and load cycling. Generally it is concluded that Eq. (2.3) provides a conservative estimation of the chord rotation at failure of RC piers.

Test results constitute the primary criterion for the development of design rules. However the lack of an adequate number of test data, especially for hollow pier sections, as well as the difficult application of test data in the design of practical applications, render determining chord rotations directly from test results practically inapplicable. For this purpose, a procedure for the determination of chord rotations at failure is adopted herein, based on an iterative procedure in accordance with the provisions of EN 1998-2 [CEN 2005b]. The results of the proposed procedure are compared with corresponding tests, in order to estimate the error of the approximation. The reason for adopting such a procedure is its simplicity and the possibility of its development in computer programs.

**Procedure for the determination of the chord rotation at failure**

The calculation of the chord rotation at failure is based on Eq. (2.3) as follows:

- The procedure adopted for the determination of the moment-curvature diagram is iterative, based on a fibre model of the section; it can be applied to any type of pier.
section (rectangular, circular, hollow, etc.).

- Section failure is considered when the reinforcement steel strain or the concrete strain reach the failure value.

- The length of the plastic hinge, \( L_{pl} \), is determined from a relation of the following form \( L_{pl} = aL + \beta d f_{sy} \), where \( d \) is the diameter of the longitudinal reinforcement bars of the section and \( f_{sy} \) is their yield stress in MPa. Coefficients \( a \) and \( \beta \) were determined via non-linear regression analysis, so as to minimize the deviation from test results, yielding the values: \( a=0.10 \) and \( \beta=0.015 \).

**Comparison with test data**

The selected procedure was applied to a series of sections that have been tested under monotonic or cycling loading up to failure, in order to determine the validity of its use for the accurate estimation of the chord rotation at failure. The most common test configuration that was used for the aforementioned tests was the cantilever, with one of the specimen ends fixed in a concrete stub. The loading was imposed as tip displacement.

The database of the tests used in this work includes 64 specimens of RC bridge pier sections under uniaxial bending with axial force. From these specimens, 31 have circular section, 8 have hollow rectangular section and 25 have rectangular section. In all tests, failure was flexural. The specimens that were included in the database were selected so that the EN 1998-2 requirements are met in terms of dimensions, reinforcement and detailing for ductility.

The procedure described in the previous paragraphs was applied for the determination of the chord rotation at failure. Figure 2.1 compares the estimated values of the plastic chord rotation at failure, \( \theta_p \), calculated by the analytical procedure and the experimental values. Agreement between the predicted and the experimental values is very good, as the low value of the standard deviation (18%) indicates. In Figure 2.1 the line of the lower characteristic value 5% that corresponds to \( \gamma_{u,\theta} = 1.25 \), as well as the line that corresponds to \( \gamma_{u,\theta} = 1.40 \) are also presented.

According to EN 1998-2, the design value \( \theta_{pd} \) of the plastic rotation \( \theta_p \) is taken as the value of \( \theta_p \) as derived from the curvature at failure divided by the safety factor \( \gamma_{u,\theta} \). This safety factor covers, apart from uncertainties due to local imperfections of the member, the scatter of test data.
Figure 2.1: Comparison between the predicted and the experimental values of the plastic rotation $\theta_p$.

## 2.1.1.3 Description of the bridges investigated and of their seismic design

The side views of the four examined bridges are presented in Figure 2.2.

For all the examined bridges, the supports of the deck at the abutments allow free longitudinal movement (sliders), rotation about the horizontal axis perpendicular to the deck and rotation about the vertical axis. Moreover for all bridges the vertical movement is fully restrained (vertical support), and the (torsional) rotation about the longitudinal axis is fully restricted.

At the abutments two cases of lateral support conditions have been examined: i) elastic support that corresponds to the “typical” transverse stiffness of the abutment and the foundation, and ii) free sliding of the deck.

The selected bridges were designed with conventional elastic seismic analysis according to AASHTO 1999 and EN 1998-2 [CEN 2005b], with the value of the behaviour factors $q$ according to the provisions of these codes and the Type 1 design spectrum of EN 1998-1 [CEN 2004a] for Class C soil, with design peak ground acceleration $a_g = 0.3 \text{g}$, Soil coefficient $S = 1.15$, $T_C = 0.60 \text{ sec}$. Response spectrum analysis was performed, with the
maximum value of the response factor $q$ that is allowed by EN 1998-2 for each bridge.

Figure 2.2: Examined Bridges

For the nonlinear analysis of single-degree-of-freedom systems and multi-degree-of-freedom systems, ground acceleration time-histories have been used that are compatible with the design spectrum. For this purpose, seven recorded ground acceleration time-histories from historic earthquakes have been modified to match the design spectrum (Figure 2.3).

Previous research in the calculation of the maximum seismic displacement demand of single-degree-of-freedom systems has essentially verified the equal displacement rule for systems with elastic period $T_e$ longer than the transition period, $T_c$, between the constant acceleration and the constant pseudo-velocity regions of the elastic spectrum.

2.1.1.4 Evaluation of the bridge response

The evaluation of the seismic response of the bridges is performed using the static
nonlinear analysis (push-over) according to Annex H of EN 1998-2 [CEN 2005b]. The evaluation of the analysis results is performed by comparing them with the corresponding results of the nonlinear dynamic time-history analysis for the ground motions specified above, which are compatible with the elastic design spectrum that was used for the seismic design of the ductile elements of the bridges. The two nonlinear analyses (nonlinear static and nonlinear dynamic) are based on the relevant guidelines of EN 1998-2, more specifically paragraphs 4.2.4 and 4.2.5, as well as Annex H for the nonlinear static analysis and E for the material constitutive laws.

![Figure 2.3: Mean acceleration response spectrum for the 7 semi-artificial accelerograms](image)

The static nonlinear analyses (pushover) and dynamic nonlinear analyses of the bridges have been performed using the computer program DRAIN-3DX. This program has been widely used by the research community for the estimation of the post-elastic seismic response of structures and incorporates an extensive element library which includes simple concentrated plasticity elements and advanced fibre model elements.

The deck was modeled with linear elastic elements. The piers were modelled with fibre model, so as to estimate their seismic response with the high accuracy considered necessary for this work, to draw general conclusions. This does not mean that simpler models cannot be used efficiently, especially for nonlinear analysis, if they describe with sufficient accuracy the nonlinear response of the elements.

In all cases the piers were considered fixed to the ground. The material laws are in accordance to Eurocodes 2 and 8. The materials are S500 steel for the reinforcement,
C35/45 for the deck and the pier concrete and Grade 270k (ASTM A416 93) for the prestressing steel.

<table>
<thead>
<tr>
<th>Yielding of Piers</th>
<th>Failure of Piers</th>
<th>Yielding of Deck</th>
<th>T-H</th>
<th>P-O</th>
<th>Elastic Design</th>
</tr>
</thead>
</table>

Figure 2.4: Analysis of the response of Bridge 1 with elastic lateral support at the abutments.

2.1.1.5 Seismic response analysis results

Lateral displacements (in meters) from the analysis are presented in Figures 2.4 to 2.11, separately for no lateral support at the abutments and for elastic support there, in the form of snapshots of the deck deformation in plan view. In particular the following results are presented:

- Deformed states of the deck as they are calculated from nonlinear static analysis that correspond to increasing values of the static seismic coefficient ($\alpha = 0.16, 0.21$ etc.)
- Deformed states of the deck corresponding to characteristic states of the piers:
  - The yielding of each pier
  - The design failure of the pier that fails first
- The deformed state of the deck corresponding to the displacement demand from an elastic response spectrum analysis.
- The deformed state of the deck at yielding in the deck itself (from transverse bending about vertical axis only for the case of elastic support of the deck at the abutments)
- The displacements of the pier heads corresponding to pier design failure condition.
- The maximum displacements of the pier heads from the nonlinear dynamic analysis.
Figure 2.5: Analysis of the response of Bridge 1 without lateral support at the abutments

Figure 2.6: Analysis of the response of Bridge 2 with elastic lateral support at the abutments

Figure 2.7: Analysis of the response of Bridge 2 without lateral support at the abutments
Guidelines for Displacement-Based Design of Buildings and Bridges

Figure 2.8: Analysis of the response of Bridge 3 with elastic lateral support at the abutments

Figure 2.9: Analysis of the response of Bridge 3 without lateral support at the abutments

Figure 2.10 Analysis of the response of Bridge 4 with transverse supports at the abutments
2.1.1.6 Conclusions

The following conclusions are derived with respect to the analysis of the deformation capacity of ductile piers:

- The deformation capacity of typical bridge pier sections can be predicted with adequate accuracy from analytical procedures published in the literature.

- The curvature of piers at yielding, $\phi_y$, remains almost constant, in the typical range of values of the normalized axial force $\nu = N_{Ed}/f_{cd}A_c = 0.10$ to $0.20$ and the longitudinal reinforcement ratio in the range of $\rho = 1\%$ to $4\%$. The approximate relations that are suggested in Annex C of EN 1998-2, provide good approximation of the yield curvature with the ratio of actual value to approximate value having a mean value of 1.1 and a standard deviation of 18%.

- The curvature at failure depends significantly on the normalized axial load $\nu$, and the longitudinal reinforcement ratio $\rho$. For low reinforcement ratios ($\rho \geq 1\%$) failure occurs because of the longitudinal reinforcement reaching its ultimate elongation. In that case large values of the curvature ductility are reached. For higher reinforcement ratios failure occurs at the confined concrete compression zone. In general for this case the ultimate curvature (and therefore the corresponding ductility) is significantly reduced, as the longitudinal reinforcement ratio increases. Moreover, in general the ultimate curvature is significantly reduced, as the normalized axial load increases. A consequence of the aforementioned conclusions is that the minimum confinement reinforcement should be increased, when the normalized axial load or the longitudinal reinforcement ratio $\rho$ increase.

- The minimum confinement reinforcement required by EN 1998-2 to avoid buckling
of the reinforcement bars in compression provides in general adequate confinement.

- By comparing the results from the application of the proposed procedure for the estimation of the design value of the ultimate displacement and the test results of a database of 64 pier specimens from literature, a remarkable agreement of predicted and experimental values is observed. In particular the standard deviation for the mean value of $\theta_p$ is 18%, whereas the lower characteristic value (5%) corresponds to a value of the safety factor $\gamma_{u,\theta} = 1.25$. This means that the proposed value of $\gamma_{u,\theta} = 1.40$ incorporates an adequate safety margin (approximately 1.12) for local imperfections.

The following conclusions are derived from the results of the parametric nonlinear analysis of the bridges studies and have general interest for bridge design:

- For all of the performed nonlinear analysis, the maximum values of the behaviour factor $q$ that are specified by EN 1998-2 were adequate for the design of bridge piers with ductile behaviour. For all the examined cases there are significant safety margins between the design value of the ultimate displacement and the corresponding maximum seismic displacement demand.

- For all the examined cases there is a remarkable agreement between the maximum displacements of the nonlinear dynamic time-history analysis and the displacements of the nonlinear static analysis that were calculated by the application of the equal displacements rule between the nonlinear system and the linear system, according to the procedure proposed in this work and EN 1998-2.

- In all cases examined, the static nonlinear analysis, for one of the two examined patterns of the lateral load (i.e. uniform distribution and proportional to the transverse mode), produced displacements at the positions of the piers that were in remarkable agreement with the corresponding maximum displacements of the nonlinear time-history analysis.

- The ratio of the shear force calculated from the nonlinear static analysis to the one calculated from elastic modal response spectrum analysis varies in the range of 1.20 to 1.64. For the piers that are critical for the design, this ratio varies between 1.46 for bridges with the deck supported at the abutments in the transverse direction and 1.54 for bridges without transverse support. These values are significantly larger than the overstrength factor $\gamma_o = 1.35$ that is proposed in EN 1998-2. It should be noted that the nonlinear static analysis procedure according to EN 1998-2 is performed with the probable values of the material strength, that is with $f_{cm}$ for the concrete and $f_{ym}$ for the steel. Taking this into account, the value $\gamma_o = 1.35$ of the overstrength factor is satisfactory for the capacity design of RC elements. However, the value $\gamma_o = 1.35$ does not appear to be conservative enough for the capacity design of the foundation.

- In the case of bridges without transverse support at the abutments, the transverse
seismic displacements of the deck at the bridge ends range from 0.8m to 3.0m for the
four examined bridges. These displacements are practically unattainable by the typical
pavement joints. The longitudinal reinforcement demands are doubled for bridges 1,
2 and 4, as compared to the corresponding demands when the deck is fully supported
at the abutments in the transverse direction. In the case of bridge 3, where the
transverse stiffness of the deck is relatively small due to length of the bridge (260m),
the reinforcement demand for the tall pier M2 is increased from 1.6% to 2.2%. The
transverse reinforcement is also increased proportionally.

- In the case of bridges with transverse support at the abutments, the displacement
curve of the deck that corresponds to the development of yielding in the deck itself is
significantly larger than the target displacement. Therefore this ultimate limit state
was not critical for the examined bridges.

2.1.2 Displacement-based adaptive pushover analysis for bridges

2.1.2.1 Introduction

Whilst the application of pushover methods in the assessment of building frames has
been extensively verified in the recent past, nonlinear static analysis of bridge structures
has been the subject of only limited scrutiny [Isakovic and Fischinger, 2006]. Since
bridges are markedly different structural typologies with respect to buildings,
observations and conclusions drawn from studies on the latter cannot really be
extrapolated to the case of the former, as shown by Fishinger et al. [2004], who
highlighted the doubtful validity of systematic application of standard pushover
procedures to bridge structures.

The parametric study has considered two bridge lengths (four and eight 50 m spans), with
regular, irregular and semi-regular layout of the pier heights, and with two types of
abutments; (i) continuous deck-abutment connections supported on piles, exhibiting a
bilinear behaviour (type A bridges), and (ii) deck extremities supported on pot bearings featuring a linear elastic response (type B bridges). The total number of bridges is therefore twelve, as shown in Figure 2.12, where the label numbers 1, 2, 3 characterise the pier heights of 7 m, 14 m and 21 m, respectively.

Since the nonlinear response of structures is strongly influenced by ground motion characteristics, a sufficiently large number of records needs to be employed so as to bound all possible structural responses. The employed set of seismic excitation, referred to as LA, is thus defined by an ensemble of 14 records selected from a suite of historical earthquakes scaled to match the 10% probability of exceedance in 50 years (475 years return period) uniform hazard spectrum for Los Angeles [SAC, 1997]. The ground motions were obtained from California earthquakes with a magnitude range of 6 to 7.3, recorded on firm ground at distances of 13 to 30 km.

The Finite Element package used in the present work, SeismoStruct [SeismoSoft, 2006], is a fibre-element based program for seismic analysis of framed structures, which can be freely downloaded from the Internet. The program is capable of predicting the large displacement behaviour and the collapse load of framed structures under static or dynamic loading, duly accounting for geometric nonlinearities and material inelasticity. Its accuracy in predicting the seismic response of bridge structures has been demonstrated through comparisons with experimental results derived from cyclic and pseudo-dynamic tests carried out on large-scale models [Casarotti and Pinho, 2006].

### 2.1.2.2 Parametric investigation: analyses and results post-processing

The response of the bridge models is estimated through the employment of (i) Incremental Dynamic Analysis (IDA), (ii) Force-based Conventional Pushover with uniform load distribution (FCPu), (iii) Force-based Conventional Pushover with first mode proportional load pattern (FCPm), (iv) Force-based Adaptive Pushover (FAP) and (v) Displacement-based Adaptive Pushover (DAP).

Results are presented in terms of the bridge capacity curve, i.e. a plot of a reference point displacement versus total base shear, and of the deck drift profile. Following Eurocode 8 [CEN, 2005b] recommendations, the independent damage parameter selected as reference is the displacement of the node at the centre of mass of the deck: each level of inelasticity (corresponding to a given lateral load level or to a given input motion amplitude) is represented by the deck centre drift, and for each level of inelasticity the total base shear $V_{base}$ and the displacements $\Delta$ at the other deck locations are monitored.

The “true” dynamic response is deemed to be represented by the results of the IDA, which is a parametric analysis method by which a structural model is subjected to a set of
ground motions scaled to multiple levels of intensity, producing one or more curves of response, parameterized versus the intensity level [Vamvatsikos and Cornell, 2002]. A sufficient number of records is needed to cover the full range of responses that a structure may display in a future event. An IDA curve set, given the structural model and a statistical population of records, can be marginally summarized (with respect to the independent parameter) by the median, the 16% and 84% fractiles IDA curves.

Comparing pushover results with IDA output, obtained from “averaged” statistics and fractile percentiles of all dynamic cases, allows avoiding the unreliable influence of single outlier values, which, statistically speaking, have reduced significance with respect to the population. From this point of view, robust measures of the means of the scattered data are used which are less sensitive to the presence of outliers, such as the median value, defined as the 50th percentile of the sample, which will only change slightly if a large perturbation to any value is added, and the fractiles as a measure of the dispersion.

Results of pushover analyses are compared to the IDA median value, for the 14 records, of each response quantity of interest; pier displacements ($\Delta_i$) and base shear forces ($V_{base}$):

\[
\hat{\Delta}_{i,IDA} = \text{median}_{j=1;14} \left[ \Delta_{i,j} \right] \quad (2.4a)
\]

\[
\hat{V}_{base,IDA} = \text{median}_{j=1;14} \left[ V_{base,j} \right] \quad (2.4b)
\]

Results of adaptive pushover analyses with spectrum scaling, which thus become also record-specific, are statistically treated in an analogous way: medians of each response quantity represent that particular pushover analysis (i.e. FAP or DAP) with spectrum scaling.

The results of each type of pushover are normalized with respect to the corresponding “exact” quantity obtained from the IDA medians, as schematically illustrated in Figure 2.13, and translated in the relationships of Eqs. (2.5). Representing results in terms of ratios between the approximate and the “exact” procedures, provides an immediate indication of the bias in the approximate procedure: the ideal target value of each pushover result is always one.

\[
\overline{\Delta}_{i,PUSHOVERtype} = \frac{\Delta_{i,PUSHOVERtype}}{\Delta_{i,IDA}} \rightarrow 1 \quad (2.5a)
\]

\[
\overline{V}_{base,PUSHOVERtype} = \frac{V_{base,PUSHOVERtype}}{V_{base,IDA}} \rightarrow 1 \quad (2.5b)
\]
Moreover, normalizing results renders also “comparable” all deck displacements (i.e. all normalized displacements have the same unitary target value), and thus a bridge index $BI$ can be defined as a measure of the precision of the obtained deformed shape. Per each level of inelasticity, the bridge index is computed as the median of normalised results over the $m$ deck locations, together with the standard deviation $\delta$, to measure the dispersion of the results with respect to the median:

$$BI_{PUSHOVERtype} = \text{median}_{i=1}^{m}(\overline{\Delta}_{i,PUSHOVERtype})$$

$$\delta_{PUSHOVERtype} = \left[ \frac{\sum_{i=1}^{m} (\overline{\Delta}_{i,PUSHOVERtype} - BI_{PUSHOVERtype})^2}{m - 1} \right]^{0.5}$$

Figure 2.13 Normalised transverse deformed pattern

Figure 2.14 Graphical representation of normalised transverse displacements
The results obtained above can then be represented in plots such as that showed in Figure 2.14, where per each increasing level of inelasticity (here represented by the deck central node displacement, indicated in the horizontal axis) the bridge index BI, computed through Eq. (2.6a), is represented with black filled symbols, whilst the grey empty marks represent the values of which the BI is the median, i.e. the normalized deck displacements given by Eq. (2.6b). In this manner, it results immediately apparent the extent (in terms of mean and dispersion) to which each pushover analysis is able to capture the deformed pattern of the whole bridge, with respect to the IDA average displacement values, at increasing deformation levels.

2.1.2.3 Parametric study: results obtained

A myriad of capacity curve plots, obtained for the different pushover analyses and compared with the IDA envelopes, was derived in this parametric study, the full collection of which can be consulted in Casarotti et al. [2005] and Pinho et al. [2007]. Herein, and for reasons of succinctness, only the most pertinent observations, together with some representative plots, are included:

(i) FCPm tends to underestimate the stiffness of the bridge, mainly due to the fact that, for the same base shear, central deck forces are generally higher compared to the other load patterns, thus resulting in larger displacement at that location; FCPm capacity curve constitutes an evident lower bound, often already in the “elastic” range, where one would, at least in principle, expect a correct prediction of the response (see Figure 2.15).

(ii) On occasions, a “hardening effect” in the pushover curve occurs: once piers saturate their capacity, the elastic abutments absorb the additional seismic demand, fully transmitted by the much stiffer and elastic superstructure, thus proportionally increasing
shear response and hence “hardening” the capacity curve. This effect, observed also in the dynamic analyses, is sometimes reproduced only by the DAP procedure, as can be observed in Figure 2.15b.

(iii) The adaptive techniques show often capacity curves quite close one to the other, and very close to FCPu (Figure 2.15a). In the elastic range, and as expected, the adaptive capacity curves lie within FCPm and FCPu curve, the same often occurring also in the inelastic range (Figure 2.15a). In some case, however, adaptive load patterns lie above the FCPu capacity curve.

Examining instead the predictions of inelastic deformation patterns, the aforementioned underperformance of FCPm is further confirmed, with poor predictions of deformed shape and/or large scatter being observed (Figure 2.16a-c). On the other hand, the drift profiles obtained with adaptive methods seem to feature the best agreement with those obtained from the nonlinear time-history analyses (Figure 2.16g-l), with DAP seemingly presenting the lower scatter. It is noted that:

(i) FCPm heavily underestimates predictions of both deformed shape and base shear, featuring also excessively high BI dispersion values. This pushover modality is therefore, in the opinion of the authors, not adequate for seismic assessment of bridges.

(ii) FCPu performs rather well for regular bridges (it leads to the best predictions in this category), however its performance worsens considerably as the irregularity of the case-study structures increases.

(iii) FAP leads to averagely good predictions throughout the entire range of bridge typology (clearly with better results being obtained for regular bridges), noting however that a relatively high value of dispersion is observed in the case of semi-regular configurations.

(iv) DAP produces also averagely good results throughout the entire set of bridges of this study, featuring in particular a very high accuracy in the case of irregular bridges (most regrettably, such high accuracy is conspicuously not present in the case of regular structures). The values of dispersion are very low, independently of bridge regularity.

2.1.2.4 Final remarks

In the framework of current performance-based design trends, which require, as a matter of necessity, the availability of simple, yet accurate methods for estimating seismic demand on structures considering their full inelastic behaviour, a study has been carried out to gauge the feasibility of employing single-run pushover analysis for seismic assessment of bridges, which have been so far object of limited scrutiny, contrary to what
is the case of building frames. Within such investigation, both conventional as well as adaptive pushover methods were used to analyse a suite of bridge configurations subjected to an ensemble of seismic records. It is noted that the bridges feature particularly non-standard shapes, both in terms of pier height distribution (certainly more irregular than what is typically found in the majority of bridges/viaducts), as well as in terms of spans length (typically, the end spans tend to be slightly shorter than their central counterparts). Such bridge configurations were intentionally adopted so as to increase the influence of higher modes in the dynamic response of the structures and in this way place the numerical tools under as tough as possible scrutiny.

Figure 2.16 Prediction of the Deformed Pattern: BI and relative scatter, plotted separately per each pushover type
The results of this analytical exercise show that, while averagely along regular and irregular configurations some conventional static force-based procedures (namely, using a uniform load distribution pattern) give results comparable to adaptive methods in estimating seismic demand on bridges, the displacement-based variant of the adaptive method associates good predictions in terms of both shear and deformed shape with a reduced scatter in the results. The latter has also proved to be most effective in representing the “hardening effect” that piers’ capacity saturation can sometime introduce in the capacity curve. In other words, whereas the application of a fixed displacement pattern is a commonly agreed conceptual fallacy, the present work witnesses not only the feasibility of applying an adaptive displacement profile, but also its practical advantages, with respect to other pushover methods.

It is important to note that the inadequacy of using conventional single-run pushover analysis to assess non-regular bridges is already explicitly recognised in Eurocode 8 [CEN, 2005], where it is stated that such analysis is suitable only for bridges which can be “reasonably approximated by a generalized one degree of freedom system”. The code then provides additional details on how to identify the cases in which such condition is and is not met, and advises the use of nonlinear time-history analysis for the latter scenarios. The results of the current work seem to indicate that the use of single-run pushover analysis might still be feasible even for such irregular bridge configurations, for as long as a displacement-based adaptive version of the method is employed. The authors feel that the latter could perhaps constitute an ideal alternative or complementary option to (i) the use of nonlinear dynamic analyses, (ii) the adoption of multiple-run pushover methods or (iii) the use of alternative definitions of reference point and force distributions, all of which might also lead to satisfactory response predictions for irregular bridges, as shown in [Isakovic and Fischinger, 2006].

Finally, it is re-emphasised that the scope of this paper was confined to the verification of the adequacy with which different single-run pushover techniques are able to predict the response of continuous span bridges subjected to transverse earthquake motion. The employment of such pushover algorithms within the scope of full nonlinear static assessment procedures is discussed in [Casarotti and Pinho, 2007; Pinho et al., 2007].
2.2 TOOLS FOR ESTIMATION OF DISPLACEMENT AND DEFORMATION DEMands IN BRIDGES

2.2.1 Simple estimation of secant-to-yield stiffness of concrete piers on the basis of test results

2.2.1.1 Introduction
In both force- and displacement-based seismic design, realistic estimation of seismic demands requires knowledge of the elastic stiffness of the structure considered as a system having bilinear force-deformation curve. For plastic hinging in the piers, this means knowledge of the pier secant-to-yield stiffness.

Test results on piers are limited and rather recent (especially on piers of large cross section, for which the effect of shear may be important). The available test results on circular or hollow piers have been collected and utilised to develop or calibrate relatively simple, yet reliable and accurate tools for the estimation of:

- the yield moment of the pier section, based on first principles, for the calculation of the theoretical effective stiffness of piers from Eq. (2.11) in Section 2.2.1.2 of Part I;
- the pier chord rotation at yielding of the pier end section, also for use in Eq. (2.11) of Section 2.2.1.2 of Part I;
- the secant-to-yield stiffness of piers for the analysis.

2.2.1.2 Moment and curvature at yielding of the pier
The yield moment, for use in the calculation of the theoretical effective stiffness of piers from Eq. (2.11) in Section 2.2.1.2 of Part I, can be determined from the plane sections hypothesis, supplemented with the material \( \sigma-\varepsilon \) laws and additional criteria as follows:

Circular piers

The yield moment is calculated on the basis of:

1. An elastic-perfectly plastic \( \sigma-\varepsilon \) law for steel,
2. A parabolic \( \sigma-\varepsilon \) diagram for concrete up to the compressive strength \( f_c \) at a strain \( \varepsilon_{\text{cr}} = 0.002 \), followed by a horizontal (rectangular) branch up to \( \varepsilon_{\text{c}} = 0.003 \).
3. A yield criterion consisting of the following, whichever occurs first:
   - yielding of the reinforcement over one-third of the part of the perimeter that falls within the tension zone;
- attainment of $\varepsilon_c = 0.003$ at the extreme compression fibre.

The neutral axis depth satisfying force equilibrium over the section (along with the yield curvature $\phi$) can be determined only iteratively. $M_y$ is obtained from moment equilibrium.

**Hollow rectangular piers**

Eqs. (2.12)-(2.18) in Section 2.2.1.2 of Part I can be applied for the calculation of the yield curvature, $\phi_y$, and the yield moment, $M_y$, taking symmetric reinforcement: $\rho_1 = \rho_2$, and the full width of the compression flange as width $b$. If the so-computed neutral axis depth, $\xi_d$, exceeds the thickness of the compression flange, then calculation of $M_y$, $\phi_y$ and $\xi_d$ takes into account the inverted-U shape of the compression zone.

Table 2.1 gives statistics of the ratio of the experimental yield moment to the value predicted as described above for circular or hollow rectangular piers. The median value of (about) 1.0 for circular piers results from the calibration of the yield criterion to yielding of the reinforcement over one-third of the perimeter in the tension zone. The overshooting by a median factor of 1.075 for hollow rectangular piers is because the section is presumed to yield at 1st yielding of the extreme tension reinforcement. Factor 1.075 should be applied as correction factor on the value of $\phi_y$ computed from Eqs. (2.13)-(2.18) of Part I.

**Table 2.1: Mean, median* and coefficient of variation of ratio of experimental-to-predicted pier properties at yielding.**

<table>
<thead>
<tr>
<th>Pier property at yielding</th>
<th>No. tests</th>
<th>mean*</th>
<th>median*</th>
<th>coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{y,exp}/M_{y,pred.-1st-principles}$</td>
<td>266</td>
<td>1.015</td>
<td>1.005</td>
<td>13.2%</td>
</tr>
<tr>
<td>$M_{y,exp}/M_{y,pred.-1st-principles}$</td>
<td>154</td>
<td>1.085</td>
<td>1.075</td>
<td>12.5%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.7)}$</td>
<td>239</td>
<td>1.05</td>
<td>0.995</td>
<td>32.4%</td>
</tr>
<tr>
<td>$\theta_{y,exp}/\theta_{y,Eq.(2.7)}$</td>
<td>139</td>
<td>1.06</td>
<td>1.015</td>
<td>29.2%</td>
</tr>
<tr>
<td>$(M_yL_s/3\theta_y)<em>{exp}/(M_yL_s/3\theta_y)</em>{Eq.(2.7)}$</td>
<td>232</td>
<td>1.07</td>
<td>1.015</td>
<td>32.1%</td>
</tr>
<tr>
<td>$(M_yL_s/3\theta_y)<em>{exp}/(M_yL_s/3\theta_y)</em>{Eq.(2.7)}$</td>
<td>139</td>
<td>1.04</td>
<td>0.98</td>
<td>35.3%</td>
</tr>
<tr>
<td>$(M_yL_s/3\theta_y)<em>{exp}/EI</em>{eff,Eq.(2.8)}$</td>
<td>232</td>
<td>1.07</td>
<td>1.015</td>
<td>31.2%</td>
</tr>
<tr>
<td>$M_yL_s/3\theta_y_{exp}/EI_{eff,Eq.(2.8)}$</td>
<td>139</td>
<td>1.085</td>
<td>1.00</td>
<td>43.1%</td>
</tr>
</tbody>
</table>

*If the sample size is large, the median represents better the central tendency than the mean, as the median of the ratio predicted-to-experimental value is the inverse of the median of the ratio experimental-to-predicted value, whereas the mean of both is typically greater than the median.
2.2.1.3 Chord rotation and theoretical effective stiffness at yielding of the pier

The following models have been fitted to the experimental value of \( \theta_y \), again taken at the corner of a bilinear \( M-\delta \) curve fitted to the envelope of the measured \( M-\delta \) loops:

- for circular piers:
  \[
  \theta_y = \phi_y \frac{L_s + a_v z}{3} + 0.0022 \cdot \max\left(0, 1 - \frac{L_s}{6D}\right) + a_{sd} \frac{\phi_y d_v f_y}{8\sqrt{f_c}} (f_y, f_c \text{ in MPa}) \tag{2.7a}
  \]

- for hollow rectangular piers:
  \[
  \theta_y = \phi_y \frac{L_s + a_v z}{3} + 0.0012 + a_{sd} \frac{\phi_y d_v f_y}{8\sqrt{f_c}} (f_y, f_c \text{ in MPa}) \tag{2.7b}
  \]

In Eqs. (2.7), \( \phi_y \) is the “theoretical” yield curvature according to Section 2.2.1.2 above, including the correction factor of 1.075 for hollow rectangular piers; \( L_s \) is the shear span (moment-to-shear ratio) at the yielding end of the pier; \( a_v \) is a zero-one variable: \( a_v = 0 \) if \( V_{Re} > V_M = M_y/L_s \) and \( a_v = 1 \) if \( V_{Re} \leq V_M = M_y/L_s \), with \( V_{Re} \) taken according to [CEN 2004b] and Eq. (2.21) in Section 2.2.1.3 of Part I; \( z \) is the internal lever arm, taken equal to \( z = 0.9D \) for circular piers and to \( z = d-d_1 \) for hollow rectangular ones; \( d_b \) is the diameter of vertical bars; \( a_{sl} \) is a zero-one variable, with \( a_{sl} = 1 \) if slippage of the vertical bars from their anchorage beyond the end section is possible, or \( a_{sl} = 0 \) otherwise.

Table 2.1 gives an overview of the fitting of the “experimental” chord rotation at yielding by Eqs. (2.7), as well as of Eq. (2.11) in Section 2.2.1.2 of Part I using the values of \( M_y, \theta_y \) obtained according to the present section, to the “experimental” effective stiffness (which is the main target of this endeavour).

2.2.1.4 Empirical effective stiffness at yielding of the pier

A purely empirical alternative to Eqs. (2.7), which is independent of the reinforcement ratio and hence convenient for estimation of the secant-to-yield stiffness before dimensioning of the pier vertical reinforcement, was fitted to the same data. If \( (EI)_c \) denotes the stiffness of the uncracked gross concrete section, the alternative is:

\[
(EI)_{eff} = \alpha \left( 0.8 + \log_{10} \frac{L_s}{h} \right) \left( 1 + 0.048 \min \left( \frac{N}{A_v}, 50 \text{MPa} \right) \right) \left( 1 - 0.25 a_{sl} \right) (EI)_{gross} \tag{2.8}
\]

where:

\( \alpha = 0.16 \) for circular piers;
$a = 0.118$ for hollow rectangular piers;

$a_{sl} = 1$ if slippage of longitudinal steel from its anchorage zone beyond the end section is possible, or $a_{sl} = 0$ otherwise.

As shown by the statistics in Table 2.1, for circular piers the empirical effective stiffness from Eq. (2.8) is overall as satisfactory as the theoretical one.

### 2.2.2 Calculation models/procedures for the secant-to-yield stiffness and equivalent damping of concrete piers, based on numerical analysis calibrated against tests

The seismic assessment and design of reinforced concrete bridges requires an accurate description of the stiffness and energy dissipation characteristics of the bridge piers when both refined and simplified methods are used to estimate performance in terms of displacements. The purpose of Section 2.2.2 is to describe the stiffness and energy dissipation properties of RC bridge piers of rectangular hollow section, for which at present insufficient information is available, based on parametric analyses calibrated against results from experimental tests performed on large-scale specimens.

Displacement-based design of bridges using simplified methods based on a linearised model of the structure requires the determination of equivalent properties of the nonlinear system. In the following, equivalent properties based on secant stiffness and energy dissipation at maximum displacement are determined for RC rectangular hollow sections. This involves the use of a continuous non-linear model of the section calibrated against experimental tests, which is used to perform parametric analyses to determine bilinear moment-curvature envelopes and energy dissipation curves in terms of the ductility of the section. The equivalent properties of a generic pier are then derived from the section properties using the plastic hinge approach. A complete description of the procedure and results obtained from the analysis is found in Paulotto et al. [2007].

The analysis starts from the identification of the parameters that play a major role in determining the behaviour of the pier sections and their ranges of variation. The following parameters were chosen:

- The section aspect ratio, $H/B$, where $H$ and $B$ denote the height and width of the section, respectively.
- The mechanical properties of the reinforcing steel and concrete.
- The longitudinal reinforcement ratio:
\[ \rho_L = \frac{A_r}{A_c} \]  \hspace{1cm} (2.9)

where \( A_r \) and \( A_c \) are the total areas of vertical reinforcement and concrete section.

- The normalized axial force:

\[ v_x = \frac{N_{Ed}}{A_c \cdot f_{ck}} \]  \hspace{1cm} (2.10)

where \( N_{Ed} \) is the axial force corresponding to the seismic design condition, \( A_c \) is the area of the concrete section and \( f_{ck} \) the characteristic value of the concrete strength.

- The confinement level, defined through Mander’s parameter \( \lambda_c \), as suggested by Annex E of EN 1998-2 [CEN, 2005b].

The range of variation of each of these parameters was determined on the basis of current practice and on prescriptions contained in the Eurocodes, in particular:

- According to EN 1998-1 [CEN, 2004a], the concrete class in primary seismic elements should not be lower than C16 in buildings of Ductility Class (DC) Medium or C20 in buildings of DC High. Concrete classes C25, C30 and C35 were considered.

- According to EN 1998-1 [CEN, 2004a], class B or C steel reinforcement, as defined in Table C.1 in EN 1992-1-1 [CEN, 2004b], should be used in primary seismic elements of buildings of Ductility Class (DC) Medium. Tempcore B500B reinforcing steel, which belongs to class B, was considered.

- From a survey of a number of bridge designs it was observed that the thickness of the walls of rectangular hollow sections varies between 0.3m and 0.5m. A constant value of 0.4m was chosen for the wall thickness.

- Vertical reinforcement ratios between 0.005 and 0.04 were considered, distributed in two layers as commonly observed in practice. Moreover, it was assumed that the rebars, all of the same size, are uniformly distributed across the section.

- According to design practice, normalized axial force values between 0.1 and 0.4 were considered.

- Values of \( \lambda_c \) ranging between 1.0 and 2.0 were considered.

The values used in the parametric analysis are summarized in Table 2.2. As a result of this preliminary analysis, 2700 possible section designs were considered.
Table 2.2: Values of the parameters used in the analysis of the pier section behaviour

<table>
<thead>
<tr>
<th>Parameter values</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall thickness [m]</td>
<td>0.40</td>
</tr>
<tr>
<td>Concrete C25 C30 C35</td>
<td></td>
</tr>
<tr>
<td>H/B</td>
<td>1.0 1.5 2.0 2.5 3.0</td>
</tr>
<tr>
<td>steel Tempcore B500B</td>
<td></td>
</tr>
<tr>
<td>$\rho_{t}$</td>
<td>0.005 0.010 0.020 0.030 0.040</td>
</tr>
<tr>
<td>$\nu_{k}$</td>
<td>0.10 0.20 0.30 0.40</td>
</tr>
<tr>
<td>$\lambda_{c}$</td>
<td>1.0 1.2 1.3 1.4 1.6 1.8 2.0</td>
</tr>
</tbody>
</table>

To obtain the moment-curvature envelope of all the sections considered, nonlinear finite element analyses (using 2D fibre models) with monotonically increasing curvatures were carried out. These envelopes, representing the capacity curves of each section, were approximated with bilinear curves to be used either for evaluation or design purposes. With this aim, the first yield point and the point corresponding to failure were evaluated for each nonlinear envelope curve. The first yield point corresponds to the point on the moment-curvature relationship at which either the first steel fibre reaches the yield strain in tension or the extreme compression fibre attains a strain of 0.002, whichever occurs first. The failure point is reached when either the longitudinal reinforcement or the confined concrete reaches its ultimate strain or when the section strength decreases down to 80% of its maximum value. The ultimate compressive strain of confined concrete is, according to Informative Annex E of EN 1998-2 [CEN, 2005b]:

$$\varepsilon_{um,c} = 0.004 + \frac{1.4 \cdot \rho_{t} \cdot f_{yw} \cdot \varepsilon_{yw}}{f_{cm,c}}$$  \hfill (2.11)

where $\rho_{t}$ is equal to twice the transverse reinforcement ratio, $f_{yw}$ and $\varepsilon_{yw}$ are the mean values of the yield stress and elongation at maximum stress of transverse reinforcement, respectively, and $f_{cm,c}$ is the mean value of the compressive strength of confined concrete.

The line that joins the origin and the first yield point gives the initial slope of the bilinear curve; the line that extends through the failure point and balances the areas between the actual and the idealized moment-curvature relationships beyond the first yield point gives the slope of the second branch (see Figure 2.17). Having defined the bilinear moment-curvature relationship, the capacity curve of a generic section may be represented through four parameters: yield curvature and moment, $\chi_{y}$ and $M_{y}$, and ultimate curvature and moment, $\chi_{u}$ and $M_{u}$, which were used to summarize the results of the parametric analysis in a series of charts [Paulotto et al., 2007].
Figure 2.17Bilinear approximation of the nonlinear envelope curve of the pier section behaviour

An example of these charts is shown in Figure 2.18 where results are expressed in terms of the following dimensionless parameters:

\[
\chi = \frac{\chi_y}{\chi_y} \quad (2.12)
\]

(ultimate curvature ductility)

\[
\mu_u = \frac{\chi_u}{\chi_y} \quad (2.13)
\]

(dimENSIONLESS yield moment)

\[
\frac{M_y}{f'_c B H^2} \quad (2.14)
\]

(post-yield hardening ratio)

\[
\alpha = \frac{\chi_u - \chi_y}{M_y} \quad (2.15)
\]

where \(f'_c\) is the mean value of the concrete compressive strength according to Table 3.1 of EN 1992-1-1 [CEN, 2004a].

By fitting these numerical results, analytical expressions can be determined to estimate the yield curvature and moment for different sections. For example, for sections with C25 concrete and Tempcore steel, the following were derived:

\[
\chi_y = 0.00552 \cdot \frac{\lambda_y}{H} \quad (2.16)
\]

and
For the two remaining parameters, \( \mu_u \) and \( \alpha \), it was not possible to derive closed form expression, however, some general considerations regarding their behaviour can be extrapolated (see Section 3.1.2 below): \( \mu_u \) decreases with increasing \( H/B \), \( \rho_L \) and \( \nu_k \) (for \( \lambda_c \leq 1.4 \)), and \( \alpha \) never exceeds 0.01 (i.e., elastoplastic), except when \( \lambda_c = 1.0 \).

The hysteretic energy dissipated by the considered sections was evaluated through nonlinear analyses under increasing cyclic curvature, calibrated against experimental results from Pinto et al. [1995, 1996]. The results of these analyses are expressed in terms of a dimensionless parameter:

\[
\eta = \frac{W}{2 \cdot \pi \cdot M_{\text{max}} \cdot \chi_{\text{max}}}
\]

(2.18)
where $W$ is the energy dissipated in one cycle, $M_{\text{max}}$ and $\chi_{\text{max}}$ are the maximum moment and curvature cyclic amplitude, respectively.

![Graph](image)

Figure 2.19 Dimensionless energy dissipated by the pier section for cycles with different ductility
The results indicate that $\eta$ does not depend on the section aspect ratio, while it depends strongly on the normalized axial force, although this dependence becomes less strong as the longitudinal reinforcement ratio increases. It was also found that by increasing the confinement of the section, the section ductility increases without any relevant changes in the section strength. Based on this result, all the cyclic analyses were conducted assuming $\lambda_c = 2.0$. The results for sections with C25 concrete and Tempcore steel are shown in Figure 2.19; for $\nu_k \leq 0.3$ they can be approximated, in terms of $\rho_L$, $\nu_k$ and the current curvature ductility, $\mu$, as follows:

$$
\eta = \left[1 - \frac{-\nu_k - 0.1}{78 \cdot \rho_L}\right] \cdot 0.96 \cdot \rho_L^{0.2} \cdot \left[1 - \frac{1}{\sqrt{\mu}}\right]
$$

Using the plastic hinge approach, quantities derived at the section level were used to compute the force-displacement envelope and the energy dissipation properties of the pier, used in turn to evaluate its equivalent stiffness and damping ratio, $K_{eq}$ and $\xi_{eq}$, respectively. The equivalent stiffness of a cantilever pier of height $L$ is defined as the secant stiffness $K_{eq}$ at maximum displacement:

$$
A_{\text{max}} = A_y \cdot \mu_A
$$

where

$$
A_y = \frac{\chi_y \cdot L^2}{3}
$$

and

$$
\mu_A = 1 + \left(\mu - 1\right) \left[\alpha + \frac{3L_p}{L} \left(1 - 0.5 \frac{L_p}{L}\right)\right]
$$

are the yield displacement and the ductility displacement of the pier, respectively, with $L_p$ denoting the plastic hinge length. Considering that the force-displacement envelope of the pier is assumed bilinear, the equivalent stiffness is expressed as:

$$
\begin{align*}
K_{eq} &= K_y \quad \text{when } \mu_A \leq 1 \\
K_{eq} &= \frac{1 + \alpha_A \left(\mu_A - 1\right)}{\mu_A} \cdot K_y \quad \text{when } \mu_A > 1
\end{align*}
$$
where

\[ K_j = \frac{3 \cdot M_j}{\chi_j \cdot L_j^3} \]  

(2.24)

\[ \alpha_j = \frac{1}{1 + \frac{3 L_j}{\alpha L} \left( 1 - 0.5 \frac{L_j}{L} \right)} \]  

(2.25)

are the secant-to-yield stiffness and the post-yield stiffness ratio of the pier, respectively.

The equivalent damping ratio of the pier, \( \xi_{eq} \), is calculated according to Jacobsen [1930]:

\[ \xi_{eq} = \frac{W_A}{2 \cdot \pi \cdot E_A} \]  

(2.26)

where \( W_A \) is the energy dissipated by the pier, approximated as:

\[ W_A = W \cdot L_p \]  

(2.27)

and \( E_A \) is the energy stored by the pier in a cycle with maximum ductility equal to \( \mu_d \):

\[ E_A = \frac{\chi_d L}{3} \mu_d M_j \left[ 1 + \alpha (\mu - 1) \right] \]  

(2.28)

By substituting Eqs. (2.18), (2.27) and (2.28) into Eq. (2.26), and setting \( M_{max} \) and \( \chi_{max} \) equal to \( M_j[1+\alpha(\mu-1)] \) and \( \mu \chi_n \), respectively, the following expression for the equivalent damping ratio of the member is obtained:

\[ \xi_{eq} = 3 \eta \frac{\mu}{\mu_1} \cdot \frac{L_p}{L} \]  

(2.29)

For the plastic hinge length, expressions from the literature, such as the one proposed by Priestley and Park [1987], were not used, since comparison with the test results used as reference in this research [Pinto et al. 1995, 1996] shows that they generally overestimate the plastic hinge length. All these empirical expressions had been derived from tests on specimens with concrete and rebars with specific mechanical characteristics. According to Manfredi and Pecce [1998], the key parameter that controls the plastic hinge length is the ratio between the ultimate and the yield strength of reinforcing steel: the higher is this ratio, the longer is the plastic hinge. The Priestley and Park [1987] expression, for
example, is derived based on tests conducted on columns with rebars characterized by over strength factors between 1.35 and 1.5. Since the ratio of ultimate to yield strength of the Tempcore steel is about equal to 1.2, the Priestley and Park [1987] expression cannot be applied to estimate the plastic hinge length in modern European bridge piers with Tempcore steel as reinforcement. Furthermore, the expressions in the literature do not relate the plastic hinge length with the ductility level of the critical section, contradicting experimental evidence. For all these reasons, curves similar to those in Figure 2.20 derived from experiments [Pinto et al. 1995, 1996] should be used.

As an example, the proposed procedure is applied to derive the equivalent stiffness and damping ratio of the A1 pier tested in Pinto et al. [1995] and of the medium and tall (MP, TP) piers of the B231C bridge tested in Pinto et al. [1996] (see Figs. 2.21 and 2.22). These three piers, constructed using Tempcore steel and C25 concrete ($f_{cm}=33$MPa), have an aspect ratio $H/B$ of the section equal to 2.0, with $H$ and $B$ equal to 1.6 m and 0.8 m respectively, and a normalised axial force, $\nu_k$, of 0.1. Table 2.3 lists other characteristics of these three piers, showing the values of the parameters used to derive the equivalent properties. The length of the plastic hinge was derived from the curves in Figure 2.20.

Table 2.3 Parameters used to derive the equivalent stiffness, $K_{eq}$, and damping ratio, $\xi_{eq}$, of the A1 pier in Pinto et al. [1995] and of the medium and tall pier (MP, TP) of the B231C bridge in Pinto et al. [1996]

<table>
<thead>
<tr>
<th></th>
<th>A1 pier</th>
<th>MP pier</th>
<th>TP pier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_e$ [m]</td>
<td>2.8</td>
<td>5.6</td>
<td>8.4</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.009</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>1.24</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>0.0033</td>
<td>0.0033</td>
<td>0.0033</td>
</tr>
<tr>
<td>$\chi_y$ [mrad]</td>
<td>3.84</td>
<td>3.81</td>
<td>3.81</td>
</tr>
<tr>
<td>$M_y$ [kN-m]</td>
<td>3558</td>
<td>4300</td>
<td>4300</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\mu_u$</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>$\Delta_y$ [m]</td>
<td>0.0100</td>
<td>0.0398</td>
<td>0.0896</td>
</tr>
<tr>
<td>$L_{pl}$ [m]</td>
<td>0.40</td>
<td>0.39</td>
<td>0.49</td>
</tr>
<tr>
<td>$K_y$ [MN/m]</td>
<td>127</td>
<td>19.3</td>
<td>5.71</td>
</tr>
<tr>
<td>$\mu_{\Delta y}$</td>
<td>7.1</td>
<td>4.3</td>
<td>3.9</td>
</tr>
</tbody>
</table>

(1) $L_{pl}$ is the plastic hinge length at the plateau of the bilinear curves shown in Figure 2.20
(2) $\mu_{\Delta y}$ is given by Eq. (2.20) when $\mu = \mu_u$
Figure 2.20: Ratio of the plastic hinge length to the pier height, in terms of the ductility level of the pier critical section and the pier aspect ratio. The proposed multi-linear curves are based on the experimental results of the A1 pier tested in Pinto et al. [1995] and of the medium and tall pier (MP, TP) of the B213C bridge tested in Pinto et al. [1996].

Figure 2.21: Equivalent stiffness in terms of pier ductility: Values obtained numerically through the proposed procedure compared to experimental ones for the A1 pier in Pinto et al. [1995] and for the medium and tall pier (MP, TP) of the B213C bridge in Pinto et al. [1996].
Figure 2.22: Equivalent damping in terms of pier ductility; Values obtained numerically through the proposed procedure compared to experimental ones for the A1 pier in Pinto et al. [1995] and for the medium and tall pier (MP, TP) of the B213C bridge in Pinto et al. [1996]
3 ESTIMATION OF COMPONENT FORCE AND DEFORMATION CAPACITIES IN BRIDGES

3.1 FORCE AND DEFORMATION CAPACITY OF CONCRETE PIERS UNDER CYCLIC LOADING

3.1.1 Simple rules for the estimation of the flexure- or shear-controlled cyclic ultimate deformation of concrete piers, on the basis of test results

3.1.1.1 Flexure-controlled ultimate chord rotation under cyclic loading

Ultimate curvature and ultimate chord rotation from the plastic hinge length

Lacking experimental measurements of curvatures at flexure-controlled ultimate conditions in circular RC piers, it is presumed that the material parameters (for confined or unconfined concrete and for the available elongation of tension reinforcement) fitted in Section 3.1.3.2 of Part I to measured ultimate curvature, \( \phi_u \), of RC members with rectangular compression zone (typical of building construction) and to the associated fixed-end rotation, all apply to circular RC piers as well. On the basis of these material parameters, the ultimate curvature, \( \phi_u \), of circular piers is determined on the basis of the plane sections hypothesis, supplemented with the following material \( \sigma - \varepsilon \) laws:

- an elastic-linearly-strain hardening \( \sigma - \varepsilon \) law for steel;
- a parabolic \( \sigma - \varepsilon \) diagram for concrete up to the compressive strength \( f_c \) at a strain \( \varepsilon_{co} = 0.002 \) (or \( f_{cc} \) for confined concrete), followed by a horizontal branch up to \( \varepsilon_{cu,c} \), given by Eq. (3.22) in Section 3.1.3.2 of Part I for confined concrete.

The neutral axis depth satisfying force equilibrium over the section can be determined only iteratively. Eqs. (3.1), (3.2) in Section 3.1.3.1 of Part I are used to obtain the ultimate curvature, \( \phi_u \), from the neutral axis depth.

For hollow rectangular piers, Sections 3.1.3.1 and 3.1.3.2 of Part I apply fully. If the neutral axis depth, \( \xi_{u,d} \), computed on the basis of these sections exceeds the thickness of the compression zone, an iterative algorithm is used, with the same basis as that employed for circular piers, to take into account the U shape of the compression zone.

The ultimate chord rotation, \( \theta_u \), of hollow rectangular piers can be determined on the
basis of Section 3.1.3.3 of Part I, using Eq. (3.26b) therein for the plastic hinge length under cyclic loading and Eq. (2.7b) of Section 2.2.1.3 in the present Part II for the chord rotation at yielding, $\theta$. For circular piers, the fitting to the available results of cyclic tests to flexure-controlled failure gave the following result, to be used in Eq. (3.25) in Section 3.1.3.3 of Part I, together with the outcome of Section 2.2.1.2 of the present Part II for the yield curvature, $\phi_y$, and of Eq. (2.7a) in Section 2.2.1.3 for $\theta$:

Circular piers - cyclic loading:

$$L_{pl} = 0.09L + \frac{2D}{3} \quad (3.1)$$

Table 3.1 gives statistics of the ratio of the experimental ultimate chord rotation to the one predicted as outlined above, for piers failing in flexure under cyclic loading.

**Empirical ultimate chord rotation of hollow rectangular piers**

As the fitting to the test results on flexure-controlled ultimate chord rotation of hollow rectangular piers by the model above and Eq. (3.26b) in Sections 3.1.3.3 of Part II, although optimal, is not very satisfactory, the empirical alternatives of Sections 3.1.3.4 of Part I, Eqs. (3.27), were extended to this type of RC member. This entails:

- taking for $\theta_u$ the outcome of Eq. (3.27a) in Section 3.1.3.4 of Part I, times 5/6, or
- taking for $\theta_u$ the sum of the outcome of Eq. (2.7a) in Section 2.2.1.3 for $\theta_y$, plus that of Eq. (3.27a) in Section 3.1.3.4 of Part I, times 0.77.

Rows 3 and 4 in Table 3.1 give statistics of the ratio of the experimental ultimate chord rotation to the so-predicted one, for piers failing in flexure under cyclic loading.

Another purely empirical alternative for the plastic component of the ultimate chord rotation, $\theta_{upl} = \theta_u - \theta_y$, can cover all type of RC members with rectangular compression zone, without distinction between walls, hollow rectangular piers or beam/columns. It is given by the following expression, with the elastic component, $\theta_y$, given by Eq. (2.7a) in Section 2.2.1.3 for hollow rectangular piers and by Eq. (2.11) in Section 2.2.1.3 of Part I for other types of members:

$$\theta_{upl} = a_1(1-0.525a_2)(1+6a_3)\left(1-0.05\max\left[10, \frac{h}{b_w}\right]\right)
\left(0.2\right)^{\left(\frac{\max(0.01\omega)}{\max(0.01\omega)} \frac{L}{h} \right)^{1.5}} \left(\frac{a_{mp}}{a_{mp}} \frac{L}{h}\right)^{1.25} \left(\frac{\omega}{\omega_{0}}\right)^{0.6} \quad (3.2)$$

where (cf. Section 3.1.3.4 of Part I):

- $a_1$: coefficient for the type of steel, equal to $a_1 = 0.022$ for ductile hot-rolled or for heat-treated (temppore) steel and to $a_1 = 0.0095$ for cold-worked steel;
- $a_2$: zero-one variable, equal to 0 for monotonic loading and to 1 for cyclic loading;
- $a_3$: zero-one variable for slip, equal to 1 if there can be slip of the longitudinal bars.
from their anchorage beyond the section of maximum moment, or to 0 if not;

\[ b_{w} / b \]

ratio of thickness of one web to the section depth;

\[ \nu = N / b f_{c} \]

(with \( b \) = width of compression zone, \( N \) = axial force, positive for compression);

\( \omega_{0} \): mechanical reinforcement ratio of tension and “web” longitudinal reinforcement,

\[ \left( \rho_{1} f_{y1} + \rho_{v} f_{yv} \right) / f_{c} \]

\( \omega_{2} \): mechanical reinforcement ratio of compression longitudinal reinforcement,

\[ \rho_{2} f_{y2} / f_{c} \]

\( f_{c} \): uniaxial (cylindrical) concrete strength (MPa)

\[ L_{s} / h = M / Vh \]

shear span ratio at the section of maximum moment;

\( \rho_{s} = A_{sh} / b_{w} h \)

t ratio of transverse steel parallel to the direction of loading;

\( f_{yw} \)

yield stress of transverse steel;

\( \alpha \)

confinement effectiveness factor from Eq. (3.21) in Section 3.1.3.4 of Part I;

\( \rho_{d} \)

steel ratio of diagonal reinforcement in each diagonal direction.

Table 3.1, rows 5-11, gives statistics of the ratio of the experimental ultimate chord rotation to the one predicted as outlined above, for all types of RC members with rectangular compression zone that fail in flexure. The statistics for hollow rectangular piers under cyclic loading are slightly inferior to the other empirical alternatives as far as the median is concerned, but better for the scatter. The overall agreement of Eq. (3.2) to the experimental results for all types of members is almost as good as that of the Eqs. (3.27) in Section 3.1.3.4 of Part I (cf. Table 3.3 therein, rows 7 to 18).

<table>
<thead>
<tr>
<th>Ultimate chord rotation predicted by different models for different types of members</th>
<th>No. tests</th>
<th>mean*</th>
<th>median*</th>
<th>coef. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq.(3.25)}-Part &amp; Eq.(3.1)-Part II} ) - Circular piers, cyclic loading</td>
<td>110</td>
<td>1.02</td>
<td>1.005</td>
<td>29.2%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq.(3.25),(3.26b)-Part I}} ) - hollow rectangular piers, cyclic loading</td>
<td>30</td>
<td>0.98</td>
<td>0.99</td>
<td>42.0%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, (5/6) Eq.(3.27a)-Part I}} ) - hollow rectangular piers, cyclic loading</td>
<td>30</td>
<td>0.955</td>
<td>1.015</td>
<td>34.4%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, 0.77 Eq.(3.27b)-Part I}} ) - hollow rectangular piers, cyclic loading</td>
<td>30</td>
<td>0.94</td>
<td>1.005</td>
<td>33.3%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq. (3.2)}} ) - hollow rectangular piers, cyclic loading</td>
<td>30</td>
<td>1.025</td>
<td>1.05</td>
<td>29.5%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq. (3.2)}} ) - All tests</td>
<td>1307</td>
<td>1.065</td>
<td>0.995</td>
<td>42.9%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq. (3.2)}} ) - All monotonic tests</td>
<td>295</td>
<td>1.15</td>
<td>1.015</td>
<td>53.0%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq. (3.2)}} ) - All cyclic tests</td>
<td>1012</td>
<td>1.04</td>
<td>0.995</td>
<td>38.3%</td>
</tr>
<tr>
<td>( \frac{\theta_{u,exp}}{\theta_{u, Eq. (3.2)}} ) - w/o slippage of bars from anchorage</td>
<td>211</td>
<td>1.145</td>
<td>0.995</td>
<td>50.3%</td>
</tr>
</tbody>
</table>
3.1.1.2 Shear resistance in diagonal tension or compression under inelastic cyclic deformations after flexural yielding

The models presented in Section 3.1.6 of Part I for the shear resistance in diagonal tension under inelastic cyclic deformations after flexural yielding, and in Section 3.1.7 of Part I for the cyclic shear resistance of walls or squat columns in diagonal compression, have been fitted to databases that include shear-critical circular and hollow rectangular piers. So, they are applicable to these types of RC members, as well.

For circular piers, in particular, the following apply:
- In Eqs. (3.29), (3.31)-(3.33), the depth of the cross-section, \( b \), is the pier diameter \( D \).
- The cross-section area, \( A_c \), in Eq. (3.29) is taken equal to \( \pi D_c^2/4 \), where \( D_c \) is the diameter of the concrete core centreline the circular hoops,
- In Eq. (3.29) the contribution of transverse reinforcement to shear resistance, \( V_w \), is:

\[
V_w = \frac{\pi A_{sw}}{2 s_h} f_{yw} (D - 2c)
\]

where \( A_{sw} \) is the cross-sectional area of a circular stirrup, \( f_{yw} \) is its yield stress, \( s_h \) is the centreline spacing of stirrups, and \( c \) is the concrete cover to reinforcement.

### Table 3.2: Mean*, median* & coef. of variation of ratio of experimental-to-predicted shear resistance

<table>
<thead>
<tr>
<th>Shear resistance for different failure modes</th>
<th># of data</th>
<th>mean*</th>
<th>median</th>
<th>*</th>
<th>coef. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{R,exp}/V_{R,Eq.\ (3.29)} ) - Part I - circular piers, diagonal tension failure</td>
<td>68</td>
<td>1.055</td>
<td>1.03</td>
<td>16.2%</td>
<td></td>
</tr>
<tr>
<td>( V_{R,exp}/V_{R,Eq.\ (3.29)} ) - Part I - hollow rect. piers, diagonal tension failure</td>
<td>24</td>
<td>1.03</td>
<td>0.98</td>
<td>17.4%</td>
<td></td>
</tr>
<tr>
<td>( V_{R,exp}/V_{R,Eq.\ (3.31)} ) - Part I - circular piers, diagonal tension failure</td>
<td>68</td>
<td>1.02</td>
<td>1.015</td>
<td>15.9%</td>
<td></td>
</tr>
<tr>
<td>( V_{R,exp}/V_{R,Eq.\ (3.31)} ) - Part I - hollow rect. piers, diagonal tension failure</td>
<td>24</td>
<td>1.09</td>
<td>1.05</td>
<td>16.4%</td>
<td></td>
</tr>
<tr>
<td>( V_{R,exp}/V_{R,Eq.\ (3.31)} ) - Part I - hollow rect. piers, web compression failure</td>
<td>51</td>
<td>1.035</td>
<td>1.01</td>
<td>17.2%</td>
<td></td>
</tr>
</tbody>
</table>

*See footnote of Table 2.1 for the median vs the mean for large sample size.

Table 3.2 gives statistics of the ratio of the experimental the experimental to the so-predicted shear resistance for shear-critical circular or hollow rectangular piers. Note that
Eqs. (3.29), (3.31), (3.32) have been fitted to all types of RC members, regardless of cross-sectional shape. So, the deviations of the median from 1.00 in Table 3.2 should be viewed in conjunction with those in its counterpart in Part I (Table 3.5 in Section 3.1.6 therein).

3.1.2 Models/procedures for estimation of ultimate deformations and shear capacity of concrete piers on the basis of the results of numerical analysis calibrated against tests

An important element for the displacement-based assessment and design of a reinforced concrete bridge is to have information on the deformation capacity of the piers and on the type of mechanism governing their failure. When performing the design of a bridge, the piers are detailed such that the flexural capacity of the section at the base is achieved while ensuring a sufficient level of shear strength in accordance with the rules of capacity design; the performance of the bridge is then satisfied by detailing the section to provide the required level of displacement capacity of the pier. For assessment, on the other hand, it is necessary to determine first the type of mechanism governing failure, i.e., to establish the relative lateral strengths in shear and in flexure; for members failing in shear, no additional member deformation will be available beyond failure, while for members failing in flexure, the available deformation after yielding is determined as a function of the detailing and geometry of the member and checked against the displacement demand.

Equally important to establishing strength and deformation capacities, is the determination of the deformability or stiffness of the pier resulting from the relative contributions of flexural and shear deformations. In the following two sub-sections, indications are given on how to account for all these variables, that complement the information given in Section 1.1 and Section 2.2.2 of Part II, necessary to complete the performance based design and assessment of a bridge.

3.1.2.1 Ultimate deformation

The displacement capacity of a bridge pier may be evaluated by means of several methodologies that vary in complexity and computational effort, ranging, for example, from the plastic hinge method to Finite Element Model (FEM) analysis.

FEM analysis is used, in general, when detailed and accurate information on the deformability of an element is needed at the expense of high computational cost, such as when performing assessment of very important structures or parts of a structure, or when other more simple approaches do not provide satisfactory results, such as for the case of short piers subjected to the combination of axial, shear and flexural loads. For this case, FEM programs such as ADAPTIC [Izzuddin and Elnashai, 1989] and Response 2000 [Bentz, 2000] maybe a viable option. For all other cases, such as when performing design
or assessment using simplified methods, the use of FEM analysis is not feasible and the plastic hinge method is proposed instead as a more computationally efficient and sufficiently accurate option for assessment and design.

The plastic hinge method, in general, gives good estimates of load-deformation response for members where the relative contribution of shear with respect to flexural deformation is not important, \textit{i.e.}, for members with shear span-to-depth ratios larger than 2.0~2.5. For members with low shear span-to-depth ratios where the contribution of shear deformation is relevant, the plastic hinge approach may still be used, following the approach exposed later in this section.

The expressions based on the plastic hinge method used to determine the load-deformation curves of rectangular reinforced concrete hollow piers as derived from experimental tests have already been presented in Section 2.2.2 of Part II. In the present section emphasis is given to the determination of the plastic hinge length and to the design or assessment of a bridge pier against target performance when flexural failure is the controlling mechanism.

According to the plastic hinge method, the displacement at the top of a cantilever pier of length \( L \) is computed as the sum of the contribution of the deformations from a plastic and an elastic region, defined as a function of the distribution of curvatures along the member. The curvatures along the plastic region span over a length \( L_p \) and are considered constant and equal to the curvature at the critical section of the member, while the curvatures along the elastic region decrease linearly to zero from the curvature at yield. The displacement \( \Delta \) at the top of the pier is then computed from the first moment of inertia of the curvature distribution about the top of the pier:

\[
\Delta = \frac{\chi_y \cdot L^2}{3} + \left( \chi_{\text{max}} - \chi_y \right) \cdot L_p \cdot \left( L - \frac{L_p}{2} \right) 
\]

(3.4)

where \( \chi_{\text{max}} \) and \( \chi_y \) are the maximum and the yield curvatures at the critical section of the pier, respectively, and \( L_p \) is the plastic hinge length. Eq. (3.4) considers that the behaviour of the pier is elasto-plastic (post-yield stiffness ratio \( \alpha \) of the section equal to zero), which is a valid approximation for bridge piers reinforced with Tempcore steel and detailed to undergo plastic deformations (\textit{i.e.}, \( \lambda > 1 \)). The length of the plastic hinge may be computed from the results of experimental tests by solving Eq. (3.4), such that for the maximum curvature measured at the base of the pier at different ductility levels, the corresponding maximum displacement measured at the top of the pier is obtained; the yield curvature is assessed from the experimental moment-curvature diagram of the section. Following this procedure and using the experimental results from Pinto \textit{et al.} [1995, 1996], the values of \( L_p \) presented graphically in Section 2.2.2 of Part II for the tall,
medium and short piers, were obtained as a function of the curvature ductility $\mu$ of the critical section, as expressed by the following expression:

$$\left\{ \begin{array}{ll}
\frac{L_p}{L} = c_L \cdot \frac{\mu - 1}{\mu L - 1} & \text{with } 1 \leq \mu \leq \mu_L \\
\frac{L_p}{L} = c_L & \text{with } \mu \geq \mu_L
\end{array} \right. \quad (3.5)$$

where $c_L$ and $\mu_L$ are two parameters that depend on the shear span-to-depth ratio $a/d$, and are equal, for the medium and tall piers ($a/d$ equal to 3.5 and 5.3), to 0.0624 and 9, and for the short pier ($a/d$ equal to 1.8), to 0.127 and 5, respectively. Since these results were derived from a very limited number of tests, it would be desirable that a larger set of experimental data is used to increase the reliability of the proposed curves.

An important feature of the procedure exposed above to calculate the length of the plastic hinge is that it allows to determine in a simplified manner the flexibility of a short pier, accounting in an empirical way for the contribution of shear deformations that otherwise would need to be obtained from more refined and computationally expensive analytical methods (See the 1st paragraph of this Section).

The displacement capacity $\Delta_u$ of a cantilever pier is computed from Eq. (3.4), by substituting $\Delta$ with $\Delta_u$ and $\chi_{\text{max}}$ with $\chi_u = \chi_\mu \cdot \mu_u$, so that for a given $a/d$ ratio (i.e., the parameters $c_L$ and $\mu_L$ are known) the displacement capacity $\Delta_u$ remains a sole function of the ultimate curvature ductility $\mu_u$ that the section can develop at the base of the pier.

The ultimate curvature ductility $\mu_u$ is obtained from the charts presented in Figure 2.18 of Section 2.2.2 of Part II, derived for reinforced concrete rectangular hollow piers as a function of the $H/B$ aspect ratio, the percentage $\rho_L$ of longitudinal steel reinforcement, the normalized axial force $\nu_k$, the confinement level $\lambda_c$ of the section (Mander’s parameter, as defined in Annex E of EN1998-2 [CEN, 2005]), and the mean values $f_{cm}$ and $f_{ym}$ corresponding to the concrete compressive and steel yield strengths. The ultimate curvature ductility $\mu_u$ was computed numerically using a fibre model and corresponds to the state when either the first concrete fibre reaches its ultimate compressive strain $\varepsilon_{cu,c}$ as determined from Eq. (2.11) of Section 2.2.2 of Part II, or when the longitudinal reinforcement reaches its ultimate strain in compression or tension, or when the strength of the section decreases to 80% of its maximum value.
Figure 3.1 shows the range of variation of $\mu_u$ for different section aspect ratios and amounts of longitudinal reinforcement for all the considered values of $\nu_k$ and $\lambda_c$ with $f_{cm}$ and $f_{ym}$ equal to 33 MPa and 575 MPa, respectively. From Figure 3.1 it can be observed that the maximum value of $\mu_u$ decreases as the aspect ratio or the amount of longitudinal reinforcement increases. This Figure is important as it gives an indication of the upper bound of $\mu_u$, which may be useful when performing a rapid screening of the deformation capacity of a bridge pier.
Figure 3.2: Proposed chart for the design of hollow rectangular piers with $\nu_k=0.10$ and $H/B = 2.0$, using displacement ductility, $\mu_u$, as performance parameter.

Figure 3.3: Proposed chart for the design of hollow rectangular piers with $\nu_k=0.10$ and $H/B = 2.0$ using drift, $\delta$, as performance parameter.

The information contained in the charts of Figure 3.2 of Section 2.2.2 of Part II may be re-arranged and expressed in such a way so as to give direct information on the displacement capacity of a pier of a given aspect ratio $H/B$ and normalized axial force $\nu_k$. This is done by plotting the variation of the ultimate curvature ductility $\mu_u$ as a function of $\lambda_c$ for different percentages of $\rho_L$, together with the field of variation of $\mu_u$ as a function of the target performance of the pier, as shown in Figure 3.2 and 3.3 for a pier section with $H/B$ equal to 2 and $\nu_k$ equal to 0.1. The target performance may be defined,
for example, in terms of a maximum displacement ductility $\mu_h$, of a maximum drift $\delta$, or of a maximum displacement $\Delta$ the pier.

If the maximum displacement ductility of the pier is used as target performance, the variation of $\mu_\epsilon$ as a function of $\mu_\Delta$ may be expressed by rearranging Eq. (3.4) and by substituting $\Delta$ with $\Delta_\mu \mu_\Delta$, and $\Delta_\mu = \chi_\mu \Delta / 3$, so that the following expression is obtained:

$$\mu_\epsilon = 1 + \frac{1}{3 \cdot \frac{L_p}{L} \left(1 - 0.5 \cdot \frac{L_p}{L} \right) \left[ \mu_\Delta - 1 \right]}$$

(3.6)

where $L_p / L$ is given by Eq. (3.5) after substituting $\mu$ with $\mu_\epsilon$. Note that for $\mu_\epsilon < \mu_c \ L_p / L$ is a function of $\mu_\epsilon$, so that Eq. (3.6) needs to be solved iteratively for $\mu_\epsilon$. The plot of Eq. (3.6) for $\mu_c$ equal to 2, 3 and 4 is given in the chart of Figure 3.2 for the medium and tall piers. Note that for a given value of $\mu_h$, $\mu_\epsilon$ is constant and independent of $\lambda_c$. The chart suggests that an increase of the section confinement (i.e., of $\lambda_c$) to achieve a larger displacement ductility capacity of the pier is most advantageous at large values of $\rho_L$.

If the maximum drift $\delta$ of the pier is used as target performance, the variation of $\mu_\epsilon$ with respect to $\delta$ is obtained by expressing $\mu_\Delta$ as:

$$\mu_\Delta = \frac{\delta \cdot L}{A_\mu}$$

(3.7)

so that by substituting Eqs. (2.21) and (2.16) from Section 2.2.2 of Part II into Eq. (3.7), which is then substituted into Eq. (3.6), the following equation is obtained:

$$\mu_\epsilon = 1 + \frac{1}{3 \cdot \frac{L_p}{L} \left(1 - 0.5 \cdot \frac{L_p}{L} \right) \left[ \frac{3}{0.00552 \sqrt{\lambda_c} \ L} \delta \cdot H - 1 \right]}$$

(3.8)

In the same way as for Eq. (3.6), Eq. (3.8) needs to be solved iteratively for $\mu_\epsilon$ if $\mu_\epsilon$ is less than $\mu_c$. The plot of Eq. (3.8) as a function of $\lambda_c$ for the non-dimensional values $\delta H / L$ corresponding to 0.02, 0.03 and 0.04 is given in Figure 3.3 for the medium and tall piers.

The use of these charts is illustrated in the following example for a section with $H/B=2$ and $v_s = 0.1$. For assessment, suppose the section is detailed with $\rho_L$ equal to 0.04 and $\lambda_c$ equal to 1.3, then the chart indicates that the maximum displacement ductility and maximum drift that the pier can develop is equal to 2.2 and 0.02 $L/H$, respectively.
Likewise, for design, suppose that the target displacement ductility is equal to 3.3 or that the target drift is equal to $0.03 \cdot L/H$, with the constraint of developing a minimum flexural strength corresponding to $\rho$, equal to 0.03, then the chart indicates that the section should be confined with a detailing corresponding to $\lambda_c$ equal to 1.48. Note that the two target performances are not equivalent (i.e., they do no lead to the same pier displacement), however, they were chosen such that for the example the same value of $\lambda_c$ would be obtained from the two charts.

For the case where the maximum displacement $\Delta$ is used as target performance, it is sufficient to substitute into Eq. (3.9) and in Figure 3.3 the term $\delta \cdot H/L$ by $\Delta \cdot H/L^2$.

In practice, for each target performance parameter, being $\mu$ or $\delta H/L$, and for each $a/d$ ratio of the pier (different parameters $\alpha$ and $\mu_s$), 20 charts are needed to consider all the combinations of $H/B$ (five values, varying from 1 to 3 at a step of 0.5) and $\nu$ (four values, varying from 0.1 to 0.4 at a step 0.1) considered in the analysis. It is foreseeable that all piers with $a/d$ in excess of 2~2.5 may be grouped under the same chart, with $\alpha$ and $\mu_s$ values similar to those derived for the medium and tall piers. In the charts where maximum displacement ductility is the target performance, $\mu$ may be varied from 1 to 5 at a step of 0.5, while for the charts where drift or displacement is the target performance, $\delta H/L$ may be varied from 0.01 to 0.045 at a step of 0.005. The dependence of these charts on the concrete compressive and steel yield strengths, as well as on the width of the pier section, should not be relevant.

3.1.2.2 Shear capacity

This section presents a brief review on the shear capacity of reinforced concrete bridge piers. The presentation is centred on the discussion of the methods available in literature to compute the capacity of tall or slender piers, and short piers.

For tall piers, where flexural deformations prevail with respect to shear deformations, the deformations of the pier are modelled through plane section behaviour, and the so-called truss models are proposed to assess the shear capacity of the member. Several truss models are reviewed, and the way that the concrete and the transverse steel reinforcement contributions to shear strength are taken into account is discussed, in particular, with reference to the effects of axial compression, displacement ductility, and shear span-to-depth ratio of the pier. A simplified method based on the Modified Compression Field Theory (MCFT) is also presented, that takes into account the interaction of flexural, shear and axial forces on the section. For the case of short piers, where the contribution of shear deformations to the total displacement of the pier is important, the strut and tie method is presented as an alternative simplified technique to compute shear strength.
The expressions presented to compute shear strength, in large part derived for rectangular and circular solid sections, are assessed against experimental results obtained from PsD earthquake tests performed on a bridge structure with reinforced concrete hollow piers.

**Truss Models**

When a reinforced concrete member is subjected to transverse loads, its shear failure may occur due to either diagonal tension failure or diagonal compression failure. Diagonal tension failure results from disruption of the load carrying mechanisms of the member (e.g., contribution of the transverse reinforcement and of the concrete in compression, aggregate interlock, dowel effect) following the formation of inclined cracks with respect to the member axis. Failure in compression results from crushing of the concrete strut that forms in the web of the member and may occur before or after the formation of inclined cracks, when either the column axial force or the transverse reinforcement ratio, or both, are relatively high, or alternatively, if the shear span ratio is relatively low.

![Figure 3.4: Ritter-Mörsch model](image)

Current design procedures for reinforced concrete members in shear stem from the original truss model proposed by Ritter [1899] and Mörsch [1909] that state that for elements with an aspect ratio greater than 2 with axial load near or below the balanced point, diagonal tension failure is the controlling mechanism. In this model it is assumed that a cracked reinforced concrete beam acts like a truss with parallel longitudinal chords and a web composed of steel ties and diagonal concrete struts inclined 45° with respect to the longitudinal axis (Figure 3.4); the tensile stresses in the diagonally cracked concrete are neglected. According to this model, when transverse loads act on a reinforced concrete member, the diagonal compressive concrete stresses push apart the loaded faces, while the tensile stresses in the stirrups pull them together. In view of these
considerations the shear resistance is computed as:

\[ V = \frac{A_v \cdot d_v \cdot f_{vy}}{s} \]  

(3.9)

where \( A_v \) is the area of shear reinforcement within a distance equal to the stirrup spacing \( s \), \( d_v \) is the effective shear depth taken as the flexural lever arm of the member and \( f_{vy} \) is the yield stress of the shear reinforcement.

Experimental tests have revealed that the results given by the model proposed by Ritter and Mörsch are generally quite conservative. In fact, the model neglects important sources of shear resistance such as aggregate interlock, dowel action of the longitudinal steel and shear carried across the uncracked concrete, as well as the effects of axial force on shear resistance.

For this reason, most construction standards and norms (e.g., ACI 318–2002, CSA - Canadian Standard Association 1994) have accepted to add an empirical correction term to the original truss equations. This term, known as the “concrete contribution”, and generally denoted as \( V_c \), is meant to represent those sources of shear resistance that the basic truss model does not take into account. With this assumption, the shear resistance can be expressed as:

\[ V = V_t + V_c \]  

(3.10)

where \( V_t \) and \( V_c \) represent the transverse reinforcement and concrete contributions to shear resistance, respectively. The transverse reinforcement contribution is given by Eq. (3.9), while the concrete contribution is taken as the shear force corresponding to the initiation of diagonal cracking, as derived from experimental tests and expressed, for example, from the following empirical expressions as found in the ACI and CSA construction codes:

\[ \text{ACI 318 - 2002} \quad V_c = 0.166 \cdot \left(1 + \frac{P}{13.8 \cdot A_g}\right) \cdot \sqrt{f'_{c} \cdot b \cdot d_e} \quad \text{(MPa)} \]  

(3.11)

\[ \text{CSA 1994} \quad V_c = 0.20 \cdot \sqrt{f'_{c} \cdot b \cdot d_e} \quad \text{(MPa)} \]  

(3.12)

where \( f'_{c} \) is the compressive strength of concrete, \( b \) is the width of the member, \( d_e \) is the effective depth of the member, \( A_g \) is the gross area of the member section, and \( P \) is the axial load (positive if compressive).
When comparing Eqs. (3.11) and (3.12), it is possible to observe that only the first equation takes into account the effect of axial force on shear resistance: axial compressive loads increase the shear load at which flexural and inclined cracking initiate. The dependency of $V_c$ on the axial load may be thought as a way to account for the effects of axial loads on the shear mechanisms neglected in the Ritter-Mörsch truss model, i.e., axial compression forces generate a larger compression zone characterized by a greater shear strength; on the contrary, axial tensile forces reduce the depth of the compression zone and leading to premature yielding of the longitudinal reinforcement, which in turn rapidly destroys the aggregate interlock mechanism. It is worth noting that some codes (e.g., ACI 318-89) consider the beneficial effect of axial loads on shear resistance only for the case of axial forces coming from external sources, such as gravity loads, while for axial forces generated from self-equilibrated systems, no beneficial effects are considered.

Acknowledging the conservative results given by the Ritter-Mörsch truss model, EN 1992-1-1 [CEN, 2004b] does not use the corrective term $V_c$, instead adopts a method known as the “variable – angle truss method”\(^1\), which is based on a truss model in which the concrete struts form with the member axis an angle $\theta$ that can vary up to a value of $45^\circ$. The method recognises that due to shear mechanisms different from those considered by the Ritter–Mörsch truss model, the compressive stresses in the member web may have an inclination lower than $45^\circ$. According to this model the shear resistance of a member may be reached either for yielding of the stirrups:

$$V = \frac{A}{s} \cdot d_s \cdot f_{ys} \cdot \cot \theta \quad (3.13)$$

or for crushing of the concrete web struts:

$$V = b \cdot d_s \cdot f'_{c} / (\cot \theta + \tan \theta) \quad (3.14)$$

where $\nu$ is the strength reduction factor for concrete cracked in shear and is computed as a function of the compressive stress $f'_c$, derived from cylinder tests:

$$\nu = 0.6 \cdot \left(1 - \frac{f'_c}{250}\right) \quad (f'_c \text{ in MPa}) \quad (3.15)$$

Eq. (3.15) accounts for the lower compression resistance of the concrete forming the

---

\(^{1}\) A combination of the variable-angle truss and concrete contribution methods, known in literature as the modified truss model approach, has been proposed by CEB [1978] and Ramirez and Breen [1991].
struts with respect to that derived from standard cylinder tests. This reduction in resistance is due to the high tensile strains that exist in the direction normal to the struts and to the mechanical disturbance caused by the stirrups crossing the struts.

The angle $\theta$ is computed such that shear resistance is attained when yielding of the shear reinforcement and crushing of the web concrete struts are reached simultaneously, leading to the following equation:

$$\cot \theta = \sqrt{\frac{1 - \omega_s}{\omega_v}}$$  \hspace{1cm} (3.16)

expressed as a function of the mechanical percentage of web reinforcement $\omega_v$:

$$\omega_v = \frac{A_f \cdot f_y}{s \cdot b \cdot v \cdot f_y}$$  \hspace{1cm} (3.17)

EN 1992-1-1 [CEN, 2004a] also allows to choose a value of $\cot \theta$ between 1 and 2.5 ("recommended" values) and to use as shear resistance of a member the lowest value resulting from Eqs. (3.13) and (3.14). For members subjected to axial compressive forces, the same code suggests to multiply the value given by Eq. (3.14) by a factor $\alpha_c$ ranging between 0 and 1.25, depending on the mean compressive stress $\sigma_p$ acting on the section of the member. EN 1992-1-1 [CEN, 2004a] does not consider any distinction between the source of the loads inducing the compressive stresses, which may be either external (i.e., gravity loads) or internal due to prestressing or posttensioning. It should be noted that EN 1992-1-1 [CEN, 2004a] does not give any indication on how to account for the effects on shear resistance associated to loads inducing tensile stresses on the section.

Both of the approaches presented in the previous paragraphs (e.g., concrete contribution and the variable angle truss method) do not take into account the reduction in shear resistance as the deformation of the section increases or after several cycles of load reversal. In view of the difficulty of modelling the deterioration of mechanisms such as tension stiffening, aggregate interlock and dowel action with the increase of member deformations, a number of codes reduce or even neglect the concrete contribution term. For example, in the case of bridges subjected to seismic actions, EN1998-2 [CEN, 2005b] suggests to assume a value of $\theta$ equal to 45° when designing plastic hinge regions for shear, i.e., the Ritter-Mörsch truss model is adopted with no concrete contribution. For bridge piers with shear span-to-depth ratios less than 2, EN1998-2 [CEN, 2005b] calculates the shear strength based on the verification of the pier against diagonal tension and sliding failure in accordance with EN1998-1 [CEN, 2004a].
In other codes, the concrete contribution reduction depends on the value of the compressive stress: if it is less than a small fraction of $f'_c$, it is set equal to zero, otherwise it is taken as a fraction of its value derived from static tests, as given for example by Eqs. (3.11) and (3.12). Unfortunately, experiments have shown that the estimate of shear strength resulting from these equations is over-conservative at low values of displacement ductility demand and under-conservative at high values of displacement ductility.

The dependence of shear strength on deformation demand has been acknowledged as early as 1975 in a comprehensive study of RC columns subjected to large displacement reversals [Wight and Sozen, 1975]. In 1983, the Applied Technology Council [ATC, 1983] published guidelines for seismic retrofit of bridges in which a conceptual model was proposed to model the relationship between shear demand and supply at different ductility levels (see Figure 3.5), which has inspired some of the contemporary approaches to model the concrete contribution term in the guidelines of design codes.

Most of the proposed models consider a constant initial value for $V_c$ up to a displacement ductility of 1 [Wong et al., 1993; Lehman et al., 1996] or 2 [Ang et al., 1989; Priestley et al., 1994], decaying linearly to a residual value at large displacement ductilities. Lehman et al. [1996], for example, set the residual value of $V_c$ equal to zero for displacement ductilities above 4.

![Figure 3.5: Applied Technology Council Model for shear strength degradation](image)

In the experimental studies carried out by Ang et al. [1989], Aschheim and Moehle [1992] and Wong et al. [1993] it has been observed that the concrete contribution is enhanced by an increase in the amount of shear reinforcement. This behaviour is represented by the models proposed by Ang et al. [1989], where the concrete residual shear strength is proportional to the amount of transverse reinforcement, and by the model of Aschheim and Moehle [1992], where the overall concrete shear contribution increases with the amount of transverse reinforcement. In the experimental studies by Ang et al. [1989] and
Wong et al. [1993] it was also observed that when the flexural ductility increases to values above two, the inclination of the diagonal compression struts of the truss mechanism with respect to the longitudinal axis decreases ($\theta < 45^\circ$), thus increasing the shear carried by the transverse reinforcement and hence that of the overall truss.

Two models, Priestley et al. [1994] and Sezen and Moehle [2004], that take into account the effects of axial load and ductility demand on shear strength, are hereafter discussed.

According to Priestley et al. [1994], the shear strength of columns subjected to cyclic lateral loads results from the summation of three contributions: concrete, $V_c$; a truss mechanism, $V_s$; and an arch mechanism, $V_p$:

$$V = V_c + V_s + V_p$$

(3.18)

The concrete component is given by:

$$V_c = k \cdot \sqrt{f'_c \cdot (0.8 \cdot A_k)}$$

(3.19)

in which the parameter $k$, defined in Figure 3.6 for plastic end regions, depends on the member displacement ductility demand. Outside the column plastic hinge region, the concrete component is computed with the value of $k$ corresponding to a ductility demand of one. The model assumes that crack opening leads only to degradation of the load-carrying capacity of concrete, with no associated degradation of the reinforcement.

![Figure 3.6: Relationship between ductility and strength of concrete shear-resisting mechanisms (adapted from Priestley et al. [1994])](image)

The contribution of transverse reinforcement to shear strength is based on a truss mechanism with an angle $\theta$ equal to $30^\circ$. For rectangular columns this contribution is:
\[ V'_v = \sqrt{\frac{3}{2}} \cdot \frac{A_h \cdot f_y \cdot D'}{s} \]  

(3.20)

and for circular columns:

\[ V'_v = \frac{\pi}{2} \cdot \sqrt{\frac{3}{2}} \cdot \frac{A_h \cdot f_y \cdot D'}{s} \]  

(3.21)

where \( A_h \) is the area of one hoop leg; \( s \) is the spacing of the layers of stirrups or hoops along the member axis; \( A_v \) is the total area of transverse reinforcement in a layer in the direction of the shear force; \( D' \) is the core dimension to the stirrup or hoop centreline.

The shear strength enhancement resulting from axial compression is considered as an independent component of shear strength, resulting from the contribution of the diagonal compression strut shown in Figure 3.7:

\[ V'_v = P \cdot \tan \alpha \]  

(3.22)

For a cantilever column, \( \alpha \) is the angle formed between the column axis and the strut extending from the point of load application to the centre of the flexural compression zone at the column plastic hinge critical section. For a column in reverse or double bending, \( \alpha \) is the angle between the column axis and the line joining the centres of flexural compression at the top and bottom of the column.

More recently, Sezen and Moehle [2004] have proposed a shear strength model for lightly
reinforced concrete columns similar to the one introduced by Priestley et al. [1994], where the $k$ factor is applied to both the concrete and truss contributions. Sezen and Moehle [2004] recognize that the damage of concrete leads to a loss of anchorage of the transverse reinforcement and to a reduction in the bond capacity of the longitudinal and transverse reinforcement, thus reducing the strength of the truss mechanisms. Based on these assumptions and considering a truss model with $\theta$ equal to $45^\circ$, the shear strength is expressed by the following equation:

$$V' = V_s' + V_c' = k_s \cdot \frac{A_s \cdot f_y \cdot d}{s} + k_t \cdot \frac{0.8 \cdot A_s \cdot 0.5 \cdot \sqrt{f_y^t}}{a/d} \cdot \left[1 + \frac{p}{0.5 \cdot \sqrt{f_y^t} \cdot A_s}\right] (\text{MPa}) \quad (3.23)$$

for shear span-to-depth ratios, $a/d$, comprised between 2 and 4. The $k$ factor is defined equal to 1.0 for displacement ductilities less than 2 and equal to 0.7 for displacement ductilities exceeding 6, varying linearly for intermediate displacement ductilities.

**Modified Compression Field Theory (MCFT)**

The compression field theory (CFT) is a procedure for the shear design of reinforced concrete members that determines the inclination angle $\theta$ of the compressive struts in a truss model following the principles of the tension field theory developed by Wagner [1929]. Equilibrium conditions, compatibility conditions and stress-strain relationships for both the reinforcement and the diagonally cracked concrete are used in the CFT to predict the load-deformation response of a member subjected to shear. The CFT methods that account for the tensile strength of concrete are known as Modified Compression Field Theory (MCFT) methods [Vecchio and Collins, 1986], [Bhide and Collins, 1989], [Collins and Mitchell, 1991].

Collins and Mitchell [1991] have proposed a simplified hand-based design method derived from MCFT, which has been adopted by a number of codes, such as the Ontario Highway Bridge Design Code [1991], the Norwegian Code [1992], the Canadian Standards Association Concrete Design Code [2004] and the AASHTO LRFD [2004] specifications.

Although Vecchio [1999] and Palermo and Vecchio [2003] have shown that the MCFT can be used to model the effects of reversed cyclic loads on reinforced concrete members, the AASHTO LRFD [2004] specifications use the MCFT design approach only for static actions; for seismic actions, AASHTO LRFD [2004] adopts a model similar to that proposed by Priestley [1994].
Hereafter, the simplified MCFT procedure adopted by CSA [2004] to compute the amount of transverse reinforcement necessary to design a section subjected to bending, axial and shear forces, $M_a, N_a$ (negative if compressive) and $V$, is briefly presented. The methodology is applicable if the shear force demand satisfies the following condition:

$$\frac{V}{\phi} < 0.25 \cdot f'_c \cdot b \cdot d$$

(3.24)

where $\phi$ is equal to 0.9.

The first step of the design procedure consists in the calculation of the longitudinal strain $\varepsilon$ at mid-depth of the section (see Figure 3.8, assuming $\cot \theta = 2$ and $\varepsilon = \varepsilon_t / 2$):

$$\varepsilon = \frac{M_a/d_a + 0.5 \cdot N_a + V}{2 \cdot (E_s \cdot A_s + E_c \cdot A_c)} \geq -0.2 \cdot 10^{-3}$$

(3.25)

where $A_s$ and $A_c$ are the areas of longitudinal reinforcement and concrete on the tension side of the beam, and $E_s$ and $E_c$ are the steel and concrete Young’s modulus, respectively; the term $A_c$ is equal to zero for $\varepsilon \geq 0$. The concrete contribution is calculated as:

$$V_c = 0.0830 \cdot \beta \cdot \sqrt{f'_c \cdot b \cdot d} \quad \text{(MPa)}$$

(3.26)

where:

$$\beta = \frac{0.0830 \cdot \phi \cdot (\cot \theta + 2)}{0.25 \cdot f'_c \cdot b \cdot d}$$

(3.27)
and $s_\text{eq}$ is the equivalent crack spacing equal to 305mm.

The portion of shear strength supplied by the shear reinforcement is given by:

$$V_s = \frac{V_u}{\phi} - V_c$$

and that the required amount of shear reinforcement is computed as:

$$\frac{A_s}{s} = \frac{V_s}{f_{yv} \cdot d_s \cdot \cot \theta}$$

with:

$$\theta = 29 + 7000 \cdot \varepsilon_c \text{ (degrees)}$$

The amount of shear reinforcement must be not less than the code minimum:

$$A_{s,\text{min}} = 0.0830 \cdot \sqrt{f_{yv} \cdot \frac{b \cdot f}{f_{yv}}} \text{ (MPa)}$$

Although not explicitly envisaged by CSA [2004], it is possible to assess the shear strength $V_s + V_c$ of a member designed with transverse reinforcement $A_s$ spaced at a distance $s$ and subjected to bending and axial forces $M_u$ and $N_u$ by iterating on a predicted value of $V_s$ until convergence on $V_s$ is reached.

When performing assessment, the amount of transverse reinforcement may be lower than the code minimum. In that case $s_\text{eq}$ is computed from the following expression:

$$s_\text{eq} = \frac{3.51 \cdot s_\text{eq}}{1.60 + a_1} \text{ (cm)}$$

where $s_c$ is the smaller between $d_c$ and the maximum distance between layers of crack control reinforcement; $a_1$ is equal to the maximum aggregate size.

More accurate predictions of shear strength may be obtained from the exact formulation of the MCFT, by using of special purpose software as implemented by Bentz [2000].
Strut-and-tie models

The “strut and tie” model is a methodology that allows to compute the shear strength of members with low shear span-to-depth ratios based on the equilibrium of forces flowing from the point of load application to the location of load transfer. Strut and tie models are generally applied to the so-called disturbed or D-regions of members, where arch action, as opposed to beam action (i.e., plane sections remain plane) is exhibited. D-regions extend about one member depth at both ends from the concentrated loads, reactions, or abrupt changes in the section or direction of the member. The shear span-to-depth ratio for which strut and tie models should be used in favour of the “truss models” to compute the shear strength of members is between 2.0 and 2.5.

Schlaich et al. [1987] suggest two guidelines in selecting a workable strut-and-tie model:

- The compatibility of deformations may be approximately considered by orienting the struts and ties within 15° of the force systems obtained from a linear elastic analysis of uncracked members and connections.
- The most valid model tends to be one which minimizes the amount of reinforcement, since this corresponds to the minimum strain energy solution.

Marti [1985] recommends three rules when using strut-and-tie models:

- Draw truss models to scale.
- Visualize the force flow using consistent equilibrium considerations.
- Ensure that truss member forces can be developed and transferred at the required locations.

More details about the strut-and-tie approach can be found in the ASCE-ACI Committee 445 on shear and torsion [1998].

Comparison against test results

The use of the proposed expressions to compute the shear strength of a reinforced concrete rectangular hollow bridge pier is given in the following; the results are compared with those obtained from the tests performed on the B213C bridge tested by Pinto et al. [1996]. Only the expressions corresponding to the truss models and the MCFT are considered and compared with the experimental results obtained from the tall and medium piers of the tested bridge.

The dimension of the cross section of the piers is 1.6 x 0.8 m, with a wall thickness of 16
cm. The flanges are each detailed with longitudinal reinforcement corresponding to 14-φ14 + 6-φ12 mm bars ($A_s = 2833$ mm$^2$, area of longitudinal steel in tension); the webs are detailed with transverse reinforcement corresponding to a total cross section of 4-φ5 mm bars ($A_v = 78.5$ mm$^2$). For evaluation purposes, the compressive strength of concrete $f_c$ and the yield strength of steel $f_y$ are taken as their mean values, equal to 33 MPa and 575 MPa, respectively. The same mean yield strength is considered in the analysis for both the longitudinal and the transverse steel, in spite of the higher yield strength, on the order of 700 MPa, obtained from tests performed on 5 mm diameter specimens of the transverse reinforcement. The normalized axial force $\nu_k$ is equal to 0.1.

The shear strength of the piers is computed at the base and at a height of 1.5 m, which differ on the amount of transverse reinforcement (60mm and 80mm, respectively) and on the level of bending moment to which they are subjected. No reduction factors on the materials or on the overall capacity of the section are used for calculating the shear strength.

The results from the tests show that the maximum shear capacity of the tall and medium piers was attained at a global ductility of 3.9 and 4.75, respectively; failure of the piers, in both cases, was controlled by flexure.

The results of the comparison of experimental and analytical results are summarised in Table 3.3. They show that the strength computed by all expressions is always higher than that obtained from the tests, thus confirming that failure occurred in flexure. Although the tests do not give any information on the actual shear strength of the member, and thus do not allow inferring on the accuracy of the proposed expressions, important conclusions can be derived by comparing the relative strengths. For example, the most conservative measures of strength are given by EN1998-2 [CEN, 2005b] and CSA [2004], while Sezen and Moehle [2004] and in particular, Priestley et al. [1994], give higher estimates of the available shear strength. The higher values of strength given by the last two expressions are due to the higher contribution of concrete, increased by the effect of the compressive axial force. In addition, the steel contribution in Priestley et al. [1994] is increased by a factor of 1.73, as the inclination $\theta$ of the concrete struts is made equal 30º.

It is also interesting to note that the expressions proposed by EN1998-2 [CEN, 2005b] and CSA [2004] lead to similar results at the critical section of the base, despite the large differences between the terms accounted by these expressions. The results obtained from this analysis and the large scatter obtained between the expressions proposed, suggests that EN1998-2 [CEN, 2005b] gives the most conservative estimate of shear strength a larger safety margin against failure in shear, which for design purposes, can be considered as beneficial. Nonetheless, research most continue in order to determine with more accuracy the range of validity of these expressions, especially when the effects of factors...
such as axial force, shear-to-span ratio and ductility demand are taken into account when performing assessment of the shear strength of a reinforced concrete bridge pier.

Table 3.3: Comparison between shear strengths computed from EN1998-2 [CEN, 2005], Priestley et al. [1994], Sezen and Moehle [2004] and MCFT CSA [2004], and the shear capacity obtained from tests on the B213C bridge by Pinto et al. [1996], for the tall and medium piers at different sections and ductility levels.

<table>
<thead>
<tr>
<th>Type of pier</th>
<th>Section description</th>
<th>Shear strength [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall, (L/H = 5.25)</td>
<td>(x=1.5 \text{ m}, s=80 \text{ mm} \mu_s=1)</td>
<td>(V_{\text{test}*})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>540</td>
</tr>
<tr>
<td></td>
<td>(x=0.0 \text{ m}, s=60 \text{ mm} \mu_s=1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(x=0.0 \text{ m}, s=60 \text{ mm}, \mu_s=3.9)</td>
<td></td>
</tr>
</tbody>
</table>

| Medium, \(L/H = 3.5\) | \(x=1.5 \text{ m}, s=80 \text{ mm} \mu_s=1\) | | | | | |
| | 800 | 2013* | 2706 | 1529 | 1163 |
| | \(x=0.0 \text{ m}, s=60 \text{ mm} \mu_s=1\) | | | 1074 | 3218 | 1828 | 1202 |
| | \(x=0.0 \text{ m}, s=60 \text{ mm}, \mu_s=4.75\) | | | 1074 | 2647 | 1451 | 1202 |

\(x\) is the distance from the base of the pier to the reference section.
* the shear capacity from tests corresponds to the ultimate capacity at maximum ductility.
# the shear capacity is computed following the provisions of EN1992-1-1

3.2 DISPLACEMENT AND FORCE CAPACITY OF ISOLATORS

3.2.1 Evaluation of displacement capacity of isolators and of the associated overstrength – effect of exceedance of isolator displacement capacity on the bridge seismic response

3.2.1.1 Introduction

In the current design practice the design working range of the isolators in terms of displacement has an upper limit defined so as to avoid sharp variation of their flexibility
that is usually triggered by a substantial change in their force resisting mechanism and may also involve certain irreversible effects. On the other hand, in the most common isolator types, the design upper displacement limit is defined by different restraint conditions, that are available beyond the design displacement capacity. These restraint conditions may be grouped in two broad types:

- **“Resistance” restraint conditions**, as in the case of low or high damping elastomeric bearings, and lead rubber bearings. For these isolators displacement restraint beyond the design displacement is affected only through the resistance of the body of the elastomeric pad.

- **“Geometric” restraint conditions**, as in the case of sliding bearings with flat or spherical sliding surface. In these isolators, displacement restraint beyond the displacement capacity is usually affected through shear-key action between steel parts.

Figure 3.9 shows typical force-displacement relations of isolators with resistance and geometric restraint conditions, respectively. It is evident that in the first type substantial displacement and force range is available beyond the design displacement of the isolator and the system failure. In the second type this range is available only in terms of force.

A third type of restraint conditions is also used, which is a combination of resistance type restraint (elastomeric or lead-rubber bearings) with a seismic link usually consisting of double shear-keys that are activated beyond the displacement capacity of the isolators.

![Figure 3.9: Typical force-displacement relations of isolators with (a) resistance and (b) geometric restraint conditions.](image-url)

The general objective of this Section is the evaluation of the definition of design displacement capacity of common isolator types within current bridge design practice and
investigation of the effects of its exceedance on bridge seismic response. The consequences of exceedance of the currently defined displacement capacity of the isolators and the possibility of profiting from the overstrength of each isolator type close to its displacement capacity is further investigated, through parametric nonlinear dynamic analyses of single-degree-of-freedom systems. Based on the results of the parametric analyses, design recommendations are proposed in order to enhance the seismic behaviour of the bridge when the displacement capacity of the isolators is exceeded.

3.2.1.2 Common types of isolators and their displacement capacity

There are two common types of isolator with large-displacement capacity: Elastomeric bearings and sliding bearings. Elastomeric bearings are available as Low Damping, High Damping and Lead Rubber Bearings. Sliding bearings are available as Friction Pendulum Bearings and Yielding Bearings. Two additional isolator types are viscous dampers acting in parallel with the other isolators of the isolating system, in order to enhance the energy dissipation of the isolating system and reduce the displacement demand. The second additional isolator type is double concave spherical bearings. This isolator type is a recent adaptation of the well-known spherical sliding bearing (or Friction Pendulum bearing).

Viscous dampers

The viscous dampers are special viscous devices, that dissipate part of the seismic energy manifested by the relative motion of the device ends. In Figure 3.10 a typical cross section of a viscous damper is shown. The device consists of a piston that moves inside a steel housing cylinder filled with a compressible silicon viscous fluid. When the two ends of the device are forced to move the piston moves and the silicon fluid is forced to pass through special orifices on the piston head. The restricted flow of the viscous fluid through the piston head orifices generates a reaction force that depends on the relative velocity $V$ between the dampers ends (viscous force). The action of the viscous damper dissipates part of the seismic input energy; this is manifested by a corresponding increase in the viscous fluid temperature.

The constitutive law of the viscous dampers has an exponential dependence on the velocity according to the following equation:

$$ F_D = C \cdot V^a $$

where $C$ is the damping constant of the device, $V$ is the relative velocity between the two ends of the device, and $a$ is a suitable exponent in the range of 0.03 to 0.40 for the typical dampers of the European industry. If the exponent $a$ is close to zero, the damper force is practically constant and independent of the velocity. For larger values of the exponent $a$ the damper force depends on the velocity. The damper force is zero at the maximum
relative displacement of the damper; therefore the damper does not contribute to the effective stiffness of the isolation system.

![Typical cross-section of a viscous damper](image)

**Figure 3.10: Typical cross-section of a viscous damper**

The displacement capacity of the viscous damper is defined by the available stroke of the piston within the housing cylinder. If this displacement capacity is exceeded, there is contact between the steel parts of the piston and the housing cylinder. This generates a sharp increase in the transmitted force by the device and acts as a “geometric” restraint condition for the isolation system. After the displacement capacity of the device is exceeded, the damper steel parts or the connections may fail, depending on their strength and the magnitude of the seismic demands.

**Double Concave Spherical Sliding Bearing**

The double concave spherical sliding bearing is a modification of the typical single concave spherical sliding bearing (or Friction Pendulum bearing). The modification consists of the addition of a second stainless-steel surface. The two stainless-steel surfaces face each other and are separated by an articulated slider, faced with a composite sliding material similar to PTFE. A typical cut view of the double concave Friction Pendulum bearing at various stages of motion is shown in Figure 3.11 [Constantinou et al. 2006]. This figure shows the concave dishes and housing plates (that are typically of ductile cast iron), the articulated slider and the sliding material (high load-low friction composite). The sliding interface plays a crucial role in the response of the Friction Pendulum Bearing. The frictional resistance of the interface is a function of the slider diameter (effecting the confinement of the composite), the contact pressure, the sliding velocity, the temperature, the wear (due to extended travel in bridge bearings because of thermal cycling). The articulated slider is required to maintain the full contact at the sliding interfaces, to accommodate differential movement along the top and the bottom sliding surfaces and prevent uneven wear of the sliding material and the stainless steel surfaces.

The hysteretic loop of single concave Friction Pendulum bearing is approximately rigid-
plastic with post-yield hardening. The actual loop is more complex, depending on a series of factors, the main one being the strong dependence on the axial force variation on the device. The actual constitutive law of the FPS element is elastoplastic with strain hardening, with a yield force and post-elastic stiffness that depend on the axial force, resulting in a hysteresis loop very much different from the standard constant shape. Other issues related for this type of device regard the variability in the friction coefficient properties due to the vertical pressure, the sliding velocity and the temperature.

![Diagram of Friction Pendulum bearing](image)

**Figure 3.11:** Cut view of the double concave Friction Pendulum bearing at various stages of motion.

All the above issues apply also to double concave Friction Pendulum bearings. When the friction coefficients of the two sliding interfaces are approximately equal, the resultant hysteresis loop corresponds to a single concave Friction Pendulum bearing with the same friction coefficient and an effective radius of curvature equal to \( R_1 + R_2 - h \), where \( R_1, R_2 \) are the radius of curvature of the two stainless steel surfaces and \( h \) is the height of the articulated slider. When the friction coefficients of the two sliding interfaces are not equal, sliding occurs only on the surface of the least friction until the force on the other surface exceeds the other friction coefficient. After that point, sliding occurs on both surfaces simultaneously. The response is characterized by a change of the post yield slope that corresponds to the initiation of sliding on both surfaces.

The displacement capacity of the Friction Pendulum bearings is defined by the available clearance between the articulated slider and the restraining ring of the stainless steel.
surface, labelled \(d\) in Figure 3.11. The displacement capacity of single concave Friction Pendulum bearings is equal to \(d\). In double concave Friction Pendulum bearings the displacement capacity is doubled (to \(2d\)) because sliding on both surfaces occurs. If the displacement capacity of the bearing is exceeded, there is contact between the articulated slider and the retainer rings and the transmitted force by the bearing increases sharply. This is like a “geometric” restraint condition for the isolation system. After the displacement capacity of the device is exceeded, the retainer ring or the articulated slider may fail, depending on their strength and the magnitude of the seismic demands.

When sliding occurs at both surfaces, the bearing behaviour is described by the effective coefficient of friction \(\mu_e\) and an effective radius of curvature \(R_e\), given by the following equations. The effect of the slider height \(h=h_1+h_2\) reduces the effective radius of curvature of the whole bearing.

\[
\mu_e = \frac{\mu_1 (R_1 - h_1) + \mu_2 (R_2 - h_2)}{R_1 + R_2 - h_1 - h_2}
\]

\[(3.34)\]

\[
R_e = R_1 + R_2 - h_1 - h_2
\]

\[(3.35)\]

The overall force-displacement relation of the double concave Friction Pendulum bearing can be obtained by considering two single concave Friction Pendulum bearings connected in series and a small mass representing the articulated slider. When the two coefficients of curvature of the surfaces are approximately equal, the overall behaviour of the bearing can be approximated as a single concave Friction Pendulum bearing with the same friction coefficient and an effective radius of \(R_e = R_1 + R_2 - h_1 - h_2\).

---

Figure 3.12: Mean Spectrum of seven semi artificial ground motions.
3.2.1.3 Analytical study

Input Motions

Non-linear dynamic analyses of the response of a single degree of freedom system (SDOF) were performed, for seven ground motions corresponding to historic records from Southern Europe or California. All these motions have been modified over duration of 15 seconds to conform to the Type 1 5%-damped elastic spectrum for Soil type C in EN 1998-1 over the whole period range examined (0.5 to 5.0s). “Design-level” seismic actions with a PGA of 0.3g were scaled at 1.0, 1.25, 1.4, 1.5, 1.6, 1.75 or 2.0 times the “design-level” earthquake. The seven ground motions are derived from well known historical records, such as that of Kalamata 1986, Friuli 1976 (Tolmezzo), Montenegro 1979 (Ulcinj2 and Herzeg Novi), Loma Prieta 1989 (Capitola), and Imperial Valley 1979 (Bonds Corner and El-Centro). The mean spectrum of the above mentioned motions, shown in Figure 3.12 along with the EN 1998-1 elastic spectrum for 5% damping.

Design Displacement Demand and Design Displacement Capacity

Regarding the relationship between the design displacement demand ($D_{bd}$) and design displacement capacity, the provisions of EN 1998-2 have been applied, i.e. it is required that the increased design displacement demand $\gamma ISD_{bd}$ does not exceed the design displacement capacity of the isolator. It is assumed here that $\gamma ISD_{bd}$ is equal to the design displacement capacity. The value $\gamma IS = 1.5$ recommended by EN 1998-2 has been used. The influence of varying this value is briefly discussed in the conclusions.

It is noted that for the considerations mentioned above the displacement is the relative displacement within the isolator. However due to the large translation stiffness of the substructure compared to that of the isolator, this displacement is practically equal to the displacement of the superstructure. This approximation becomes important for the extension of the behaviour of the devices beyond the displacement at which a stopper is activated, as in the case of viscous dampers, or the device itself behaves as a stopper, as in the case of the Friction Pendulum Bearings.

Viscous Dampers and Elastomeric Bearings

The effect of exceedance of the displacement capacity of viscous dampers is investigated for the case of an isolating system consisting of low damping elastomeric bearings and viscous dampers, which is typically applied in bridge applications. To this end, parametric time-history analyses are performed for a simplified SDOF system. The system parameters that are examined cover the range of values for typical bridge applications.
The action of the viscous dampers can be approximately modelled by an equivalent effective damping ratio of the single-degree-of-freedom system. This approximation is exactly accurate for the case of a damping exponent equal to 1.0 and yields good results for viscous devices with relatively big damping exponents.

Due to the quite narrow loops of their force-displacement diagram, low damping elastomeric bearings can be modelled with satisfactory approximation as bilinear elastic systems with 5% viscous damping. This simplified model is sufficient for the purposes of this study, as it can capture the influence of the main parameters of the problem.

The range of values of system parameters in the parametric studies is (Figure 3.13a)

- Period of the system ($T$), which defines the initial stiffness $K_1$ and covers a range of typical values for isolated bridges, i.e. 2s, 3s, 4s.
- Effective damping ratio $\xi_{\text{eff}}$ of the SDOF system that corresponds to the energy dissipation by the viscous dampers. Three damping ratios are considered: 5%, 10%, 20%, and 40%.
- Stiffness ratio ($K_2/K_1$) before and after the design displacement capacity ($\gamma_{\text{ISD}} \cdot \text{Dbd}$) which defines the stiffening of the system. The value $K_2/K_1 = 250$ is used to cover the case of the geometric restraint when the displacement capacity of the dampers $\gamma_{\text{ISD}} \cdot \text{Dbd}$ is exceeded, beyond which the system oscillates with the period of the substructure (assumed as $T_{\text{Substructure}} = 0.4s$).

![Figure 3.13: a) Simplified model for the case of elastomeric bearings and viscous dampers. When the displacement capacity $\gamma_{\text{ISD}} \cdot \text{Dbd}$ of the dampers is exceeded, the stiffness of the system is increased to $K_2$, which corresponds to the geometric restraint. The ratio $K_2/K_1$ is assumed equal to 250. b) Simplified model for the geometric restraining when the displacement capacity of the friction pendulum bearing is exceeded.](image-url)
Double concave friction pendulum bearings

The simplified model used in this study consists of two single concave Friction Pendulum bearings connected in series. Each of the single concave bearing models the corresponding concave surface of the double concave Friction Pendulum bearing. Both surfaces are assumed to have the same radius of curvature $R$. Various combinations for the coefficients of friction $\mu_1, \mu_2$ of the two surfaces are examined. For displacements of the overall bearing exceeding the design capacity of the system, $\gamma S_{db}$, a restrainer is activated (geometric restraining) that acts as a stopper between superstructure and substructure. The behaviour of the restrainer is illustrated in Figure 3.13b by the branch with stiffness $K_3$, which is assumed to be equal to the stiffness of the substructure (0.4s). This model is sufficient to capture the influence of the main parameters of the problem. The examined values for the main parameters of this model are:

- The post-yield period $T$ of the double concave Friction Pendulum bearing corresponds to the effective radius of curvature $R_e = R_1 + R_2 - h_1 - h_2$ according to the following relation:

$$T = 2\pi \sqrt{\frac{R_1 + R_2 - h_1 - h_2}{g}}$$

(3.36)

Three values are examined for the post-yield period that covers a range of typical values for isolated bridges namely 2s, 3s and 4s.

- Four combinations are considered for the coefficients of friction $\mu_1, \mu_2$ of the two surfaces: $\mu_1=0.11, \mu_2=0.11$; $\mu_1=0.08, \mu_2=0.11$; $\mu_1=0.05, \mu_2=0.11$; $\mu_1=0.02, \mu_2=0.11$.

- Stiffness of the fully restrained super structure ($K_3$) when the system oscillates with the period of the substructure ($T_{assumed}$ approximately 0.4s).

Results

The results from nonlinear analysis are presented in terms of normalised displacements, $D_{max}/D_{db}$, and normalised forces, $F_{max}/F_{bd}$, as function of the scale factor on the design seismic action. Figures 3.14-3.17 show the normalised displacements and forces for different scale factors on the input motion and two different periods $T=2.0s$ and $3.0s$. 
Figure 3.14: Viscous Dampers and Elastomeric Bearings with T=2s

Figure 3.15: Viscous Dampers and Elastomeric Bearings with T=3s

Figure 3.16: Double Concave Friction Pendulum Bearings with T=2s
3.2.1.4 Conclusions

Viscous Dampers and Low Damping Elastomeric bearings

The results show reduction of the displacement demand. This reduction starts at lower values of the scale factors ($\gamma IS=1.5$) and is more pronounced at the maximum investigated scale factor of 2.0 (reduction by approximately 35%), for systems with short period ($T=2s$) than for systems with long period ($T=4s$). For the later the reduction starts at scale factor values above 1.5 and is less pronounced (about 20%) at a scale factor of 2.0. The different values of the effective damping ratio $\xi eff$ of the viscous dampers (5%, 10%, 20%, and 40%) examined do not seem to have a significant effect on the reduction of the displacement demand. The system parameter that has a significant effect on the displacement demand is the period of the elastomeric bearings.

For combination of elastomeric bearings with viscous dampers that act as stoppers when the design displacement capacity ($\gamma ISDbd$) is exceeded, a very sharp force increase takes place at scale factor $\gamma IS = 1.50$. The increase is much higher for larger values of the scale factor. Moreover, a substantial force increase takes place in this case, even for values of the action scale factor lower than $\gamma IS = 1.50$ (starting at scale factor value of about 1.25). This is due to the fact that the seismic motions used, even after their modification to conform to the design spectrum, have a dispersion of displacement demands, that causes exceedance of design displacement capacity of the bearings and therefore activation of the stopper even for scale factors lower than $\gamma IS = 1.50$. The sharp increase of force demand is more pronounced for smaller effective damping ratios of the viscous dampers and longer periods $T$ of the elastomeric bearings.

It should also be noted that if (scaled) natural records are used as input, the acceleration spectrum of such records according to EN 1998-2 should not be lower than the design
elastic spectrum, even locally. These records would present even higher dispersion than the motions used above. Therefore the corresponding deviations would be higher.

Double Concave Friction Pendulum Bearings

The displacement demand increases or decreases with respect to the action scale factor depending on the post-yield period of the bearing. For $T = 2.0s$ there is a reduction of displacement demand by approximately 10% for an earthquake scale factor equal to 2.0, while for $T = 4.0s$ the increase is about 25%. The bilinear behaviour of the double concave Friction Pendulum bearing, when different coefficients of friction are used for the two concave surfaces, reduces the displacement demand. When the difference of the coefficients of friction of the two surfaces increases, this effect is more pronounced.

These bearings are much less sensitive in terms of force demand to the increase of the action scale factor than low damping elastomeric bearings with viscous dampers. The sensitivity is higher for systems with larger post-yield periods, whereas the use of different friction coefficients for the two surfaces does not seem to have a significant effect. Some sensitivity appears for values of the action scale factors less than $\gamma_S = 1.50$, for the reason given above in the case of low damping elastomeric bearings and viscous dampers.

Design displacement capacity of the isolators

In view of the high sensitivity of the isolators found in this study, a redefinition of the displacement capacity of common isolator types does not appear feasible. In particular, for the case of low damping elastomeric bearings it does not appear to be safe to propose an increase of the design displacement capacity with respect to the current definition. When viscous dampers are used together with elastomeric bearings, the displacement capacity of the viscous dampers should be substantially larger than the design displacement capacity of the elastomeric bearings (e.g. at 1.25 times this displacement).

For the case of sliding bearings, such as the Friction Pendulum bearings and the double concave Friction Pendulum bearings, in view of the high sensitivity of the displacement demands near the design displacement capacity, it is recommended that an additional margin of displacement on the order of 20% is provided.

Design recommendations to enhance the seismic behaviour of the bridge at large displacements

When the force demand exceeds the ultimate load capacity of the isolator, failure of the system is governed by the isolator. This may occur by failure of the isolator itself, or of its anchorage to the substructure or superstructure. Note EN 1998-2 requires the isolator
and its anchorage to be designed for the increased design displacement $\gamma ISD_{bd}$.

Design practices that enhance the seismic behaviour of the bridge when the displacement capacity of the isolators is exceeded include the design of the devices that act as geometric restraints when their displacement capacity is exceeded as sacrificial links. The sharp increase of the transmitted force when a geometric restraint is activated is not desirable for the substructure of the bridge. Therefore, the devices that act as a geometric restraint when their capacity is exceeded may be designed to fail shortly after their capacity is exceeded. The connections of the viscous dampers or the body of the device may be designed to fail shortly after the transmitted force exceeds the value corresponding to their displacement capacity. The force increase of the elastomeric bearings when their capacity is exceeded is much smaller and the transmitted forces are reduced significantly.

This design cannot be easily applied to Friction Pendulum bearings, because the articulated slider may fall outside the concave sliding surface if the retainer ring fails when the displacement capacity is exceeded. Special detailing may be applied for this case, such as a level concrete surface where the slider can move when the retainer ring is broken.

**Value of amplification factor $\gamma IS$**

According to EN 1998-2, this factor multiplies the seismic displacement demands resulting from the analysis so as to obtain the required design seismic displacement capacity of the isolators. The value used for this factor in the present study is the one recommended by EN 1998-2 i.e. $\gamma IS = 1.50$. It is evident from the figures presenting the results of this study in terms of force demands, that the sensitivity of these demands would dramatically increase if a value lower than 1.50 had been used.

### 3.2.2 Displacement re-centring capacity of bridge isolation systems

#### 3.2.2.1 Introduction

The restoring capability (or re-centring capacity) is identified by the current design codes as a fundamental feature of seismic isolating systems [AASHTO 2000, EN1998-2, IBC2000, NEHRP 2000 etc]. However all the regulations concerning the restoring capability evaluation are not based on theoretical fundamentals but on a rather empirical approach. Systems with adequate restoring capability demonstrate a tendency to return to zero displacement position during the seismic event. Poor restoring capability is manifested by: i) substantial residual displacements, compared to the isolation system displacement capacity, ii) accumulation of displacements during a sequence of seismic events, iii) higher uncertainty in the estimation of the maximum displacement demand, and iv) increased maximum and residual displacements for asymmetric seismic inputs.
with large-amplitude pulses, typical of near-fault records.

The restoring capability of the isolating system increases by elastic restoring forces, such as the force from the rubber stiffness of elastomeric bearings and the curvature restoring force of spherical sliding bearings (e.g. the Friction Pendulum System - FPS). It decreases by hysteretic behaviour (hysteretic dampers), yielding of lead in Lead Rubber Bearing (LRB) and friction forces in sliders. The balance of these counteracting components defines the restoring capability of the isolating system.

The evaluation of restoring capability of isolating systems by examining the displacement response from a limited number of non-linear analyses can lead to misleading results, because re-centring behaviour depends strongly on the post-elastic isolating system properties as triggered by certain not easily recognized details of the ground motion.

Unlike the peak displacement, which increases monotonically as the earthquake scaling factor increases, the residual displacement depends on the earthquake scaling factor in a non-monotonic, non-systematic manner. For some values of the scaling factor, the peak displacement is large, but the residual displacement is almost zero. Moreover, for different records with similar maximum displacements, the residual displacements may differ by an order of magnitude. Therefore, in order to establish reliable conclusions, the isolating system should be analysed for a large set of earthquake records.

The general objective of Section 3.2.2 is the evaluation of the restoring capability of common types of isolating systems within the current bridge design practice. The restoring capability is evaluated in terms of residual displacements after the event. In order to produce a reliable estimate of the restoring capability, a large earthquake record database is used; many earthquake scaling factors are examined for each record. Extensive parametric studies are performed for single-degree-of-freedom (SDOF) bilinear systems with various properties corresponding to typical isolating systems and the results are processed statistically. Relations estimating the seismic residual displacements and the accumulation of residual displacements of past events are derived using nonlinear regression methods. A criterion is then proposed for the required displacement capacity in order to accommodate the accumulation of residual displacements.

### 3.2.2.2 Analytical study

**Input motions**

Non-linear dynamic analyses of SDOF systems were performed for 122 ground motions corresponding to historic records. The records encompass a wide variety of site soil conditions and distance from the fault, including records with near source effects.
Modelling of seismic isolating systems

The majority of hysteretic or frictional isolators can be simulated with bilinear hysteretic models. The bilinear model is fully defined by three independent parameters, which are typically selected for seismic isolation systems as the characteristic strength $F_0$, the post-yield stiffness $K_p$, and the yield displacement $d_y$. [Figure 3.18].

Without loss of generality, the normalized parameters $F_0/W$ and $W/K_p$ are used in the analyses. These parameters have a physical meaning for spherical sliding systems but can be used equivalently for all seismic isolating systems with bilinear behaviour, such as the LRB. The yield displacement $d_y$ is the remaining parameter to fully define the hysteresis loop (e.g. 0.125mm to 0.5mm for sliding systems and 5mm to 15mm for LRBs).

The hysteretic systems can be in static equilibrium at a displaced position with zero external force. This is achieved when the force due to $K_p$ exactly equilibrates the corresponding fraction of the force at zero displacement $F_0$. For all hysteretic systems, there is a maximum displacement at which the system can be in static equilibrium at a displaced position. This maximum static residual displacement is denoted by $d_r$ and can range from zero for elastic systems, to infinity for perfectly plastic ones. The restoring capability of an isolation system improves as $d_r$ becomes smaller. This is because the value of $d_r$ is the maximum possible value of the residual displacement. The value of $d_r$ is a function only of the properties of the isolating system. For a bilinear system, it is equal to:

$$d_r = \frac{F_0}{K_p}$$

(3.37)

The residual displacement when the system is unloaded quasi-statically from a displacement equal to the maximum earthquake displacement $d_{max}$ is defined as $d_{res}$. Obviously, the maximum residual displacement of the system following any earthquake
inducing maximum displacement equal to $d_{max}$ cannot exceed $d_{in}$ The value of $d_{in}$ cannot exceed $d_r$ or $d_{max}$. For a bilinear hysteretic system, $d_{in}$ is given by the following relation:

$$d_{in} = d_r \cdot \max \left[ 0, \min \left( 1, \frac{d_{max} - d_y}{d_r + d_y} \right) \right] \tag{3.38}$$

**Parametric study**

For each case of the parametric study presented in Table 3.4 a non-linear dynamic analysis was performed for the corresponding bilinear system and earthquake ground motion. A total of 180 different SDOF systems were examined and about 110000 non-linear dynamic analyses were performed. For each case the maximum displacement $d_{max}$ and the residual displacement $d_{res}$ were stored for further statistical processing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>No. of cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0/W$ ($\mu$)</td>
<td>0.03, 0.045, 0.06, 0.075, 0.09</td>
<td>5</td>
</tr>
<tr>
<td>$W/K_0$ ($R$)</td>
<td>2m, 3m, 4m, 5m, 6m, 100m</td>
<td>6</td>
</tr>
<tr>
<td>$d_y$</td>
<td>0.125mm, 0.250mm, 0.500mm, 5mm, 10mm, 15mm</td>
<td>6</td>
</tr>
<tr>
<td>Ground motion</td>
<td>122 recorded ground motions</td>
<td></td>
</tr>
<tr>
<td>Earthquake Scaling Factor</td>
<td>0.50, 0.75, 1.00, 1.25, 1.50</td>
<td>5</td>
</tr>
</tbody>
</table>

**Distribution of residual displacements**

In Figure 3.19 the distribution of residual displacements obtained is presented in the form of histograms for systems with different properties. The residual displacement $d_{res}$ is normalized with the residual displacement $d_{res}$ corresponding to the maximum displacement of each earthquake. Each different panel histogram corresponds to systems with different $F_0/W$ and $W/K_0$ values and includes 610 data points (122 earthquakes x 5 scaling factors). All the presented systems have the same yield displacement $d_y=5$mm. It is observed that for all systems the normalized residual displacement $d_{res}/d_{in}$ can have any value in the range of 0.0 to 1.0. However, for systems with good restoring capability (low $F_0/W$ and $W/K_0$) the most probable values of $d_{res}/d_{in}$ are close to zero, whereas for systems with poor restoring capability (large $F_0/W$ and $W/K_0$) the histogram tends to the uniform distribution and the most probable values of $d_{res}/d_{in}$ are closer to 1.0.

The distributions of the normalized residual displacement $d_{res}/d_{in}$ indicate a large scatter in
the data, which increases for systems with poor restoring capability. The large scatter is attributed to fact that the residual displacement is affected considerably by the details of the ground motion, which vary significantly. Because there is a significant deviation of the data from their mean value, it is necessary to define a design value for the residual displacement that is greater than the mean value, such as the mean plus one standard deviation or a value which is not exceeded by a certain percentage of the observed data.

A simple relation that captures the observed behaviour has the form:

\[
\frac{d_{res}}{d_{rm}} = \frac{C_0}{\left(1 + C_1 \frac{d_{rm}}{d_r}\right) \left(1 + C_2 \frac{d_{res}}{d_r}\right)} \tag{3.39}
\]
where \( C_0, C_1, \) and \( C_2 \) are coefficients to be estimated. Non-linear regression analysis is performed, to estimate the values of all the three coefficients \( C_0, C_1, \) and \( C_2 \) to achieve the best fit to the data. The same relation with different values for the coefficients \( C_0, C_1, \) and \( C_2 \) is used for the estimation of various statistical properties of the observed normalized residual displacement \( d_{\text{res}}/d_{\text{rm}} \). Because \( d_{\text{res}}/d_{t} \) depends on the value of \( d_{\text{max}}/d_{r} \) and \( d_{t}/d_{r} \), the two independent variables that will be used in the analysis are the ratios \( d_{\text{res}}/d_{t} \) and \( d_{t}/d_{r} \). The dependent variable is the normalized residual displacement \( d_{\text{res}}/d_{\text{rm}} \).

Table 3.5: Results of the regression analysis for the estimation of the statistical properties of the normalized residual displacement \( d_{\text{res}}/d_{\text{rm}} \).

<table>
<thead>
<tr>
<th>Statistical Property of ( d_{\text{res}}/d_{\text{rm}} )</th>
<th>Coefficient ( C_0 )</th>
<th>Coefficient ( C_1 )</th>
<th>Coefficient ( C_2 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.539</td>
<td>4.298</td>
<td>30.769</td>
<td>0.974</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.307</td>
<td>2.213</td>
<td>19.720</td>
<td>0.942</td>
</tr>
<tr>
<td>Median (50th Percentile)</td>
<td>0.552</td>
<td>6.180</td>
<td>41.139</td>
<td>0.967</td>
</tr>
<tr>
<td>80th Percentile</td>
<td>0.869</td>
<td>4.276</td>
<td>31.683</td>
<td>0.966</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>0.972</td>
<td>3.300</td>
<td>25.508</td>
<td>0.961</td>
</tr>
</tbody>
</table>

In order to estimate particular statistical properties of the normalized residual displacement \( d_{\text{res}}/d_{\text{rm}} \), the independent variables \( d_{\text{max}}/d_{r} \) and \( d_{t}/d_{r} \) are grouped in bins with width equal to 0.10 and 0.02, respectively. The values of the normalized residual displacement \( d_{\text{res}}/d_{\text{rm}} \) for the data points within each combination of the independent variables bins are processed statistically. Only bins with 100 or more data points are considered for an adequate statistical sample. The statistical properties of interest are the mean value, the standard deviation, the 80% fractile and the 90% fractile. The proposed relation is fitted to the data, to estimate the values of the coefficients \( C_0, C_1, \) and \( C_2 \) for each of the aforementioned statistical properties of \( d_{\text{res}}/d_{\text{rm}} \). Table 3.5 shows the results of the non-linear regression analysis. Values of \( R^2 \) greater than 0.94 illustrate excellent fit for all the statistical properties of \( d_{\text{res}}/d_{\text{rm}} \).

The proposed relations give constant values of residual displacement \( d_{\text{res}} \) when the maximum earthquake displacement \( d_{\text{max}} \) is larger than (about) \( d_{r} \). This is obvious, because \( d_{\text{res}}/d_{\text{rm}} \) is constant and equal to \( d_{r} \) when \( d_{\text{max}}>d_{r}+2d_{r} \). This leads to the important conclusion that the estimated residual displacement is constant for all the earthquakes with a maximum displacement \( d_{\text{max}} \) larger than \( d_{r}+2d_{r} \). This can be explained because for systems with \( d_{\text{max}}>d_{r}+2d_{r} \) the residual displacement after the earthquake cannot exceed the maximum static value of \( d_{r} \) which is a property of the isolation system and independent of the maximum earthquake displacement \( d_{\text{max}} \). The validity of this conclusion is verified by the observed data points for \( d_{\text{max}}/d_{r}>1 \) that attain a constant value, independent of \( d_{\text{max}}/d_{r} \).
The coefficients of the proposed relation show that the mean values of the estimated residual displacement are very close to the median values. Moreover the mean plus one standard deviation values of the estimated residual displacement are very close to the 80% fractile values. For design purposes the values are recommended by the authors. This estimation of the residual displacement is defined as the design value of the residual displacement \( d_{rd} \) for the given isolating system and earthquake. The estimated coefficients \( C_0 \), \( C_1 \), and \( C_2 \) for the estimation of 80% fractile values lead to the following relation for the design value of the residual displacement:

\[
d_{rd} = \frac{0.869d_{rm}}{\left(1 + 4.276 \frac{d_{rm}}{d_r}\right)} \left(1 + 31.683 \frac{d_{rm}}{d_r}\right)
\]  

(3.40)

Figure 3.20 shows the estimated design residual displacement \( d_{rd} \) normalized to \( d_{max} \), as a function of \( d_{max}/d_r \) and \( d_y/d_{max} \). It is useful for the evaluation of the effect of the isolating system parameters \( d_r \) and \( d_y \) for systems with given maximum seismic displacement \( d_{max} \). The design value of the residual displacement decreases as \( d_{max}/d_r \) and \( dy/d_{max} \) increase.

The evaluation of the system restoring capability in terms of the residual displacement after the earthquake leads to the following conclusions: The key system parameters that describe the restoring capability are the maximum static residual displacement \( d_r \) and the ratio, \( d_y/d_r \). The key ground motion parameter that affects the restoring capability is the normalized maximum displacement, \( d_{max}/d_r \). The restoring capability increases as the values of \( d_{max}/d_r \) and \( d_y/d_r \) increase.
Guidelines for Displacement-Based Design of Buildings and Bridges

The evaluation of the system restoring capability in terms of the residual displacement after the earthquake leads to the following conclusions: The key system parameters that describe the restoring capability are the maximum static residual displacement \( d_r \) and the ratio \( d_y/d_r \). The key ground motion parameter that describes the restoring capability is the normalized maximum displacement \( d_{\text{max}}/d_r \). The restoring capability increases as the values of \( d_{\text{max}}/d_r \) and \( d_y/d_r \) increase.

**Effect of distance from the fault on the residual displacements**

Ground motions recorded a few kilometres from the fault plane have been observed to differ dramatically from typical far-fault records. The near-fault effects are particularly unfavourable for systems with poor restoring capability and lead to increased maximum and residual displacements.

The present analysis verifies that the increased residual displacement for systems with poor restoring capability subjected to near-fault ground motions is taken into account indirectly by the increased value of the maximum displacement \( d_{\text{max}} \) for systems with poor restoring capability under near-fault ground motions. This can be explained as follows:

The effect of the earthquake record before the maximum displacement \( d_{\text{max}} \) occurs is fully described by the value of \( d_{\text{max}} \). This is because the value of \( d_{\text{max}} \) can fully describe the system’s state at that time as the velocity and hysteretic variable of the system are known and equal to 0.0 and \( \pm 1.0 \) respectively. Therefore, the ground motion after the time when the maximum displacement \( d_{\text{max}} \) occurs is the part of the ground motion that is not captured by the value of the maximum displacement \( d_{\text{max}} \) and may modify the residual displacement. The period of the pulses that characterize near fault records is typically shorter than the effective period of the seismic isolating system. Therefore it is expected that the maximum earthquake displacement \( d_{\text{max}} \) occurs after the main pulse has ended. The part of the ground motion after the end of the main pulse has the same features as typical far-field records and is expected to affect the residual displacement in the same manner, regardless of the distance from the fault. Thus it is concluded that the proposed relations provide a good estimation of the residual displacement regardless of the distance from the fault, because normalized values of residual displacements \( d_{\text{res}}/d_{\text{rm}} \) are used.

**3.2.2.3 Proposed restoring capability criterion**

The criteria concerning the restoring capability specified by AASHTO 2000, exclude the use of certain isolating systems, independently from their deformation capacity. The corresponding criteria of EN 1998-2 specify a minimum displacement capacity of the isolating system, depending on its properties, that is considered to satisfy the necessary re-centring requirements. A similar possibility is offered also by the criteria of IBC 2000.
The criteria proposed by the present work follow the second approach (i.e. minimum required displacement capacity), because it is directly related to the effects of accumulation of residual displacements. The criteria yield increased displacement capacity for systems with poor restoring capability. Such a capacity may be larger than the required displacement capacity due to other code requirements. The proposed criteria take into account possible accumulation of residual displacements and the uncertainty in the estimation of the maximum displacement for systems with poor restoring capability.

The general form of a relation that can be used as a code provision to evaluate the restoring capability of seismic isolation systems is as follows:

\[ d_{\text{cap}} \geq \gamma R_1 d_{Ed} + \gamma R_2 d_{rd,Ed} + d_0 \]  

(3.41)

Where \( d_{\text{cap}} \) is the displacement capacity of the isolation system, \( d_{Ed} \) is the design seismic displacement, \( d_{rd,Ed} \) is the design value of the residual displacement of the design seismic action and \( d_0 \) is the offset displacement due to permanent and thermal actions. Coefficient \( \gamma R_1 \) is introduced to increase the value of the design seismic displacement, \( d_{Ed} \), in order to take into account that systems with poor restoring capability have an increased uncertainty in the estimation of the design seismic displacement \( d_{Ed} \) when the current design code procedures are used. According to EN 1998-2 the design seismic displacements of the isolating system are increased by the amplification factor \( \gamma IS \), which has a recommended value of 1.50. Coefficient \( \gamma IS \) takes into account the increased displacement demand for rare earthquake events (maximum possible earthquake) and the various uncertainties in the estimation of the design earthquake displacement. Therefore, it is reasonable to assume that the uncertainty in the estimation of the design displacement can be expressed by a value of the coefficient \( \gamma R_1 \) smaller than the recommended value of 1.50 for the coefficient \( \gamma IS \). The value of \( \gamma R_1 \) proposed here is 1.20.

Coefficient \( \gamma R_2 \) accounts for a possible accumulation of residual displacements before the design earthquake occurs. Smaller earthquakes produce smaller residual displacements than the design earthquake, but are also more frequent than the design earthquake. Accumulation of these residual displacements is probably higher than that of a single design earthquake. The value of \( \gamma R_2 \) proposed here is 1.50.

### 3.2.2.4 Comparison of the proposed relation with current code provisions

In Figure 3.21 the required displacement capacity for adequate restoring capability according to the corresponding code provisions and the proposed relation is plotted as a function of the system parameter \( F_0/W \) and the normalized design displacement \( d_{rd,Ed}/d_0 \) which is the key parameter expressing the restoring capability of the isolation system for the design earthquake. The yield displacement of the isolating system \( d_y \) is assumed to be
zero, because this leads to larger capacity demands for adequate re-centring. Moreover, the value of the displacement capacity $d_{cap}$ presented in the figures corresponds to the term $d_{cap} - d_0$, which means that only the effect of the re-centring requirements is examined.

Only the EN 1998-2 relations and the proposed relation specify a required displacement capacity to fulfill the re-centring requirements. This capacity may be larger than the required capacity of $\gamma_{SI} \cdot d_{Ed}$ from seismic analysis. The AASHTO 2000 relations cannot be satisfied by increasing the displacement capacity of the isolating system. The required capacity increase for systems that do not satisfy the restoring capability requirement of IBC2000 (3.0 times the design displacement) is also presented for comparison purposes.

![Figure 3.21: Required displacement capacity for re-centring capability according to code provisions and the proposed relation, as a function of the normalized design displacement $d_{Ed}/d_r$ and of system parameter $F_0/W$. In all cases the yield displacement is zero ($d_y=0$) and the additional displacement capacity for permanent and thermal actions $d_0$ is not included.](image-url)
With reference to Figure 3.21 the following remarks are made:

i) The proposed relation expresses a rational increase of the displacement capacity, in order to limit the accumulated residual displacements to a small fraction of the displacement capacity. Assuming a value of 1.50 for the displacement amplification factor $\gamma_{IS}$, the displacement capacity of systems with $d_{\text{Ed}}/d_r > 0.50$ is unaffected by the proposed re-centring requirement. The parametric analyses presented here show that such systems possess excellent restoring capability.

ii) The required displacement capacity for adequate re-centring is increased smoothly as the ratio $d_{\text{Ed}}/d_r$ decreases and becomes equal to 2.5 times the design displacement $d_{\text{Ed}}$ for elastic-perfectly plastic systems with $d_{\text{Ed}}/d_r = 0$. This value is close to the corresponding value of 3.0 in IBC2000. The proposed relation allows a smooth transition from systems with adequate restoring capability to those with poor restoring capability. Essentially every system with positive post-yield stiffness is permitted, provided that the displacement capacity is suitably increased if the restoring capability of the system is not sufficient.

iii) The EN 1998-2 relations express increased displacement capacity as the restoring capability of the isolating system is reduced. However the displacement capacity tends to infinity as $d_{\text{Ed}}/d_r$ goes to zero. This essentially prohibits the use of systems with small $d_{\text{Ed}}/d_r$ ratios, as the required capacity increase becomes excessively big. The EN 1998-2 relation (a) ($\Delta F_{\text{m}} \geq \delta W d_r / \delta d_{\text{m}}$) depends on the system parameter $F_0/W$. For small values of the system parameter $F_0/W$, this relations becomes excessively conservative. For typical values of the system parameter $F_0/W$ larger than 0.025, the EN 1998-2 relation (b) ($d_{\text{m}} \leq d_{\text{Ed}} - \delta d_{\text{m}}$) is critical. The required capacity increase by this relation is very conservative, as it is significant even for systems with $d_{\text{Ed}}/d_r$ ratio larger than 0.5 that have excellent restoring capability.

### 3.2.2.5 Conclusions

- Evaluation of restoring capability of isolating systems on the basis of the displacement response from a limited number of non-linear analyses can lead to misleading results. This is due to the strong dependence of re-centring behaviour on the post-elastic isolating system properties and on certain not easily recognized details of the ground motion and its non-monotonic dependence on the earthquake scaling factor.

- The key parameter describing the restoring capability of isolating systems subjected to the design earthquake is the ratio $d_{\text{Ed}}/d_r$, where $d_{\text{Ed}}$ is the design earthquake displacement and $d_r$ is the maximum possible static residual displacement. For bilinear systems, $d_r = F_0/K_p$, where $K_p$ is the post-yield stiffness and $F_0$ is the characteristic strength (force at zero displacement of the hysteresis loops). The restoring capability of the isolating system increases as the ratio $d_{\text{Ed}}/d_r$ increases.
Another system parameter that affects the restoring capability is the ratio $d_y/d_r$, where $d_y$ is the yield displacement of the isolating system. The restoring capability of the isolating system increases, as the ratio $d_y/d_r$ increases.

- For systems with $d_y/d_r$ larger than 0.5 the accumulation of residual displacements remains practically constant, independently of the maximum displacement response of the system. For such systems the proposed criterion, Eq. (3.38), concerning the required displacement capacity covering the accumulation of residual displacements is not critical, provided that the capacity of the isolating system is defined in accordance with EN 1998-2 with $\gamma_{IS}=1.50$.

- The current AASHTO 2000 restoring capability requirements do not depend on the characteristic strength $F_0$, which expresses the hysteretic component of the behaviour that leads to imperfect re-centring. On the other hand the AASHTO 2000 requirements compare the increase of the post-yield force with the weight of the structure, $W$. Therefore the AASHTO 2000 relations can be either very conservative or unsafe, depending on the magnitude of $F_0/W$. By comparing the AASHTO 2000 requirements with the proposed criterion, it is concluded that AASHTO 2000 is more conservative when $F_0/W<0.05$, or less conservative when $F_0/W>0.05$.

- The current EN 1998-2 requirements are generally very conservative, even for systems with $d_y/d_r$ larger than 0.5 that present good re-centring according to this study. The EN 1998-2 provisions require a very large increase of the displacement capacity for systems that do not have adequate restoring capability. The increase tends to infinity for elastic-perfectly plastic systems, essentially prohibiting such systems. The proposed criterion on provides a smooth transition between systems with good restoring capability and poor restoring capability. For the limit of elastic perfectly plastic systems, the required capacity increase according to the proposed criterion is in the order of 2.5 times the design displacement, which is in agreement with the relevant provision of IBC2000 (i.e. 3.0 times the design displacement).

### 3.2.3 Effect of axial force variation on seismic response of bridges isolated with friction pendulum systems

#### 3.2.3.1 Introduction

A standard friction pendulum system device (FPS) is based on the principle of the sliding pendulum motion. It consists of two sliding plates, one of them (indifferently, the bottom or the top one) is characterized by a concave spherical surface. The two plates are connected by means of an articulated friction slider and PTFE bearing material, as schematically represented in the cross section depicted in Figure 3.22a.
Typically a FPS device may provide equivalent dynamic periods of vibration in the range from 2 to 5 seconds and displacement capacities greater than 1 m. Detailed descriptions of the basic principles of the FPS devices can be found in literature in relatively recent works [Almazan et al., 2002; Wang et al., 1998; Tsai, 1997].

The FPS finite element model developed in this study is characterized by a restoring force
$V$ and by a relative displacement $\Delta z$ between the top and the bottom plates of the FPS (Figure 3.23a and Figure 3.23b) where both $V$ and $\Delta z$ are radially-directed vectors.

The proposed numerical model has been implemented in Feap, a computer code for static and dynamic non-linear analyses [Taylor, 2001]. Presently, the simulation does not include possible uplift of the deck, therefore a tensile force in the isolator is permitted. This may result in an increase of compression on the other isolator on the top of the same pier and in an increased bending moment and shear. While this aspect may be easily taken into account in future studies, its relevance will be discussed whenever appropriate.

The behaviour of the isolator is described by the $V - \Delta z$ non-linear constitutive law and its main characteristic is to be sensitive to the axial load variations $\Delta N$ (AM model); for this reason the proposed model has a variable yield point and a post-elastic stiffness that depend on the actual axial force, resulting in a non-linear post-elastic branch. It is worth underlining that the majority of the models that can be found in literature depend on the initial axial load level, but they are usually insensitive to $\Delta N$ (NAM model), that is the post-elastic branch of the constitutive law remains linear.

In Figure 3.24 the different features of the two models are shown; in particular the continuous lines are associated to the responses of the two isolators (AM model), one subjected to a progressive increase in compression and the other to a progressive decrease of axial load. On the opposite, if the NAM model is considered (dashed line), the response is insensitive to axial force variations.

![Figure 3.24](image_url)

(a) Constitutive laws of FPS models sensitive (AM) and insensitive (NAM) to axial load variations, (b) Hysteretic loops of the model sensitive to the axial load variation for different initial axial forces
3.2.3.2 Parametric study: deck geometry and bridge models

A rather extensive bibliographic investigation was carried out [Chen and Duan, 2000; IIC, 1996-2001; Leonhardt, 1979; O'Connor, 1971] in order to identify bridge deck cross-sections most commonly used in the design practice. Within the scope of this study, a parametrised deck cross-section characterized as a function of the geometrical parameter $\beta$, given by the ratio between the relative distance $d$ of two bearings and the vertical distance $H$ between the centre of gravity of the deck mass and the FPS base (Figure 3.25), is employed. This parameter controls the axial force variations $\Delta W_R$ due to the rotational equilibrium to lateral seismic load, characterised by opposite signs. A second axial force variation $\Delta W_V$ may be produced by the vertical component of the seismic input, this being characterized by the same sign on both bearings. The interested reader is referred to Calvi et al. [2004] for a detailed treatment of these matters.

![Figure 3.25 Causes of the axial force variations on the isolators at the top of the pier](image)

The straight bridge models consist of six piers of the same height, neglecting the effects of elevation irregularities. The piers were modelled by frame elements fixed at the base, characterized by linear elastic behaviour and sections, considered as cracked. Moment-curvature analyses were performed in order to calculate the flexural stiffness joining the origin to the yielding point. The deck was represented by linear elastic frame elements; the stiffness was kept constant, without considering any effect of a variation of span length and aspect ratio. No abutment was modelled, assuming that the model represents a section of a longer viaduct. Both AM and NAM models were considered to simulate the behaviour of the isolators at the top of the pier.

3.2.3.3 Parametric study: seismic input

The selection of the input ground motions to be used in this study was based on several requirements:

- The need of adopting natural records, in order to avoid possible bias related to abnormal frequency contents, number of cycle, duration and input energy;
• The necessity of a relatively high horizontal peak ground acceleration, to fully trigger the expected phenomena and to avoid large scaling that may affect the properties of the records;

• The importance of using a consistent set of two horizontal and one vertical components – more specifically the fundamental relevance of the vertical input is evident, since the focus of the study is on axial force variation effects.

• The desire to have a consistent, though relatively wide representation of magnitude, source mechanism, epicentral distance, duration, soil type.

On this basis, a set of five natural accelerograms (three components) was selected [see Calvi et al., 2004]. These accelerograms are used in the numerical analyses described in subsequent sections. All components of each accelerogram were scaled with a single factor, with a view to obtain a horizontal peak ground acceleration of 0.8 g. Typical ratios of displacement demands at corresponding periods are in the order of four, with values typically ranging between 0.2 and 0.8 m.

3.2.3.4 Parametric study: results overview

A total of 280 nonlinear time-history analyses were performed. The results confirmed that bridge geometry, isolator model and consideration of vertical component do not affect displacement demand and dissipated energy, which remain essentially constant, with variations that never exceed 10 %, as shown for an exemplificative case in Figure 3.26. As expected, the displacement demand is concentrated in the isolators, with negligible substructure displacements.

On the contrary, significant axial force variation may result on each device, as
exemplificatively shown in Figure 3.27, where the maximum values obtained in a single isolator during the response are depicted. Although these results have to be considered with some care, since a linear elastic model was used for the piers and no uplift was simulated (while in several cases the variation shown is larger than 100%), they do nonetheless lay evident that the presence of the vertical component is of the utmost importance, with negligible variations in case of responses to horizontal components only. The results are affected to a much lesser extent by a variation of the deck aspect ratio and by the model used.

![Graph](image1.png)

Figure 3.27 Coalinga, axial load variation with and without vertical input; (a) $\beta = 1.00$, (b) $\beta = 2.50$

Significant variations, of up to 70%, in transverse shear demand were also observed when vertical input was considered, again highlighting the important role played by the latter in the response of FPS-isolated bridges [see Calvi et al, 2004].

### 3.2.3.5 Case-study application: shear capacity-demand comparison

In the previous section, the potential importance of considering the effects of axial force variations on the seismic response of bridges isolated with friction pendulum systems was illustrated, showing that this may induce increased shear demands and possibly unpredicted torsional moment on the piers. To get a feeling of the possible relevance of these effects with respect to safety, it becomes necessary to consider capacity as well as demand and this obviously implies the need of designing sample pier sections, and considering the actual variation of both demand and capacity as a function of axial force.

For this purpose, a specific sample case of a straight railway bridge was considered, characterized by six piers with a constant height of 10 m, span length of 39 m and deck aspect ratio equal to 1.00, designing longitudinal and transversal reinforcement according to standard code rules and design response spectrum (as defined in Eurocode 8 for stiff soil). Consistently with the input ground motions used for the non linear analyses, a PGA equal to 0.8 g was assumed to anchor the spectra.
As a function of axial force level, bending, shear and torsional strengths of each pier were calculated at each time step of the time histories, using a standard flexural model for bending, the shear model proposed by Kowalsky and Priestley [2000], and a model proposed by Collins and Mitchell [1991] for torsion. The interaction between bending, shear and torsion [Collins and Lampert, 1971; Collins and Mitchell, 1991; Henry and Zia, 1982] was neglected, for simplicity.

The capacity-demand comparison depicted in Figure 3.28 renders apparent that a shear failure could indeed take place, therefore confirming the potential relevance of the phenomenon being object of this study.

![Figure 3.28 Base shear capacity – demand comparison along the longitudinal direction; (a) AM and (b) NAM demands, Northridge CSE earthquake](image)

### 3.2.3.6 Closing remarks

In the present work, the effect of the axial force variation on the seismic response of bridges isolated with friction pendulum systems has been investigated. A series of parametric non-linear time history analyses have been performed using five ground motion records with the transversal components scaled to a maximum PGA of 0.8 g. The horizontal and vertical components of each seismic event have been considered. An analytical model of friction pendulum device whose yielding force and post-elastic stiffness are sensitive to the axial load variations has been developed.

Although the range of parameters considered was rather ample, still some potentially significant effects have had to be overlooked. These include possible deck uplifting, bending-shear-torsion interaction, flexural nonlinear response of the pier. Therefore, the conclusions of the study have to be considered as preliminary results to be further investigated in the future.
Generally speaking, the fundamental result is that the inclusion of axial force effects may not be significant for what concerns variation of the displacement demand, but may induce important increment of shear, bending and torsional moment demand on the piers.

The fundamental parameters that may amplify, or reduce, these effects are the ratio between deck and pier mass, the aspect ratio of the deck, the intensity of the ground motion and the consideration of vertical input, as briefly discussed below.

- **Ratio between deck and pier mass**: a significant variation of the shear force transmitted from the deck to the pier may result in strongly attenuated effect at the pier base when the ratio of the pier mass to the deck mass is high.

- **Aspect ratio of the deck**: for the same level of horizontal force, the axial force variation possibly induced by the horizontal acceleration is higher for a deck section relatively larger and for devices relatively closer one to each other.

- **Intensity of the horizontal ground motion**: relatively high horizontal peak ground accelerations may induce more significant effects, such as in the case considered, where a PGA of 0.8 g is assumed.

- **Consideration of vertical input**: the inclusion of the vertical component of the input ground motion is of critical importance, since it may lead to cases where shear demand exceeds the corresponding capacity.

A fundamental aspect related to bridge design should also be noted. It has been a relatively common practice to assume that possible pier collapses are capacity protected by the shear capacity of the bridge isolation system. This implies that there is no reason to protect a possibly brittle shear collapse mode with a lower strength flexural yielding of the pier. However, as shown in this work, this situation may not apply if a significantly higher shear force is transmitted from the deck to the pier. As a consequence, it is felt that it is appropriate to recommend that when using friction pendulum systems, capacity design principles are still applied to protect undesired failure modes of the pier and foundation system.

A simple and immediate development of this work may consist of non-linear dynamic analyses with more refined models of the bridge elements (such as piers with non-linear behaviour), but also new parameters must be investigated (bridge configuration, irregularities in elevation, different deck types, framed piers, different isolator devices) in order to confirm the information obtained in this research.
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LIST OF SYMBOLS AND ABBREVIATIONS

\( a, b, c, d, e, f \) = Parameters defining the shape and the size of the loading surface and the failure criterion

\( A_c \) = cross-section area

\( b \) = width of the compression zone

\( b_c \) = width of the confined core to the centreline of the hoop

\( b_c' \) = width of the confined core to the inside of the hoop

\( b_w \) = width of rectangular web

\( C.o.V. \) = coefficient of variation

\( D \) = Diameter of the foundation

\( D \) = diameter of circular pier

\( D_{d0} \) = Additional displacement capacity of the isolating system for permanent and thermal actions

\( D_{d0} \) = Design Displacement of the Bearing

\( D_1 \) = diameter of confined core to the centreline of the hoop in a circular pier

\( d_1 \) = distance of the centre of the compression reinforcement from the extreme compression fibres

\( d_b \) = diameter of longitudinal reinforcement

\( d \) = section effective depth.

\( d_{b1} \) = diameter of longitudinal reinforcement

\( d_{cp} \) = Displacement capacity of the isolating system

\( d_{Ed} \) = Design seismic displacement of the isolating system

\( d_{ini} \) = Initial earthquake displacement of the isolating system

\( d_{max} \) = Maximum earthquake displacement of the isolating system

\( d_{res} \) = Residual displacement corresponding to the displacement capacity \( d_{cp} \) of the isolating system, i.e. residual displacement when the isolating system is unloaded quasi-statically from its displacement capacity \( d_{cp} \)

\( d_r \) = Maximum static residual displacement i.e. maximum possible residual displacement for which the isolating system can be in static equilibrium

\( d_{rd,Ed} \) = Design value of the residual displacement of the design earthquake

\( d_{rd} \) = Design value of the residual displacement of the earthquake

\( d_{res} \) = Residual earthquake displacement of the isolating system

\( d_{res,Ed} \) = Residual displacement corresponding to the design seismic displacement \( d_{Ed} \) of the isolating system, i.e. residual displacement when the isolating system is unloaded quasi-statically from the design seismic displacement \( d_{Ed} \)

\( d_{max} \) = Residual displacement corresponding to the maximum earthquake displacement \( d_{max} \), i.e. residual displacement when the isolating system is unloaded quasi-statically from the maximum earthquake displacement \( d_{max} \).

It is the maximum possible residual displacement after an earthquake with
maximum displacement $d_{\text{max}}$

d_y = Yield displacement of the isolating system
$E_c = \text{elastic modulus of concrete}$
$E_s = \text{steel elastic modulus.}$
$F = \text{Current loading point;}$
$F_0 = \text{Characteristic strength of the isolating system i.e. force at zero displacement of the hysteresis loops}$
$F_{\text{bd}} = \text{Design Force of the Bearing}$
$F_u = \text{Ultimate Force Capacity of the Bearing}$
$f_c = \text{loading surface}$
$f^{\infty} = \text{failure criterion}$
$f_u = \text{uniaxial (cylindrical) concrete strength}$
$f_c = \text{concrete compression strength}$
$f_t = \text{concrete tensile strength}$
$f_y = \text{yield stress of longitudinal reinforcement}$
$f_{\text{ty}} = \text{yield stress of transverse reinforcement}$
$G = \text{Flow rule}$
$H = \text{plastic modulus}$
$h = \text{depth of cross-section}$
$h_c = \text{depth of the confined core to the centreline of the hoop}$
$h_{\text{hi}} = \text{depth of the confined core to the inside of the hoop}$
$q_{\text{max}} = \text{Ultimate pressure of the foundation under vertical centre load}$
$q = \text{Hardening generalized array}$
$r = \text{Radius of curvature of the concave surface of the spherical sliding bearing}$
$r = \text{descending traction curve factor } r = \varepsilon_{\text{tm}} / \varepsilon_{\text{t}} \text{ in the fibre model}$
$s_b = \text{centerline spacing of stirrups}$
$T = \text{Period of the system}$
$u_{\text{el},x,y} = \text{Reduced horizontal elastic displacements}$
$u_{\text{pl},x,y} = \text{Reduced horizontal plastic displacements}$
$u_{\text{el},x,y} = \text{Reduced horizontal elastic displacements}$
$u_{\text{pl},x,y} = \text{Reduced horizontal plastic displacements}$
$V, V' = \text{Vertical force, reduced vertical force}$
$V_R = \text{shear force resistance}$
$V_w$ = contribution of transverse reinforcement to shear resistance
$x$ = compression zone depth
$Z$ = modulus of the descending part of the compression curve for unconfined concrete in the fibre model
$z$ = internal lever arm

$\alpha, \beta, \delta, \eta$ = Kinematic hardening variables for dof $(x^u, \theta^u, y^u, \varphi^u)$ respectively

$\alpha$ = confinement effectiveness factor (according to Sheikh and Uzumeri, 1982)
$\gamma$ = 2nd root on axis $V'$ of the current loading surface
$\gamma_{IS}$ = amplification factor of the design displacement of the isolating system according to Eurocode 8 – Part 2 (the proposed value is 1.50)

$\delta_{d_{max}}$ = Design value of the modification of the maximum displacement $d_{max}$ due to the initial displacement $d_{ini}$
$\delta_{d_{max}}$ = Modification of the maximum displacement $d_{max}$ due to the initial displacement $d_{ini}$
$\delta_{d_{res}}$ = Design value of the modification of the residual displacement $d_{res}$ due to the initial displacement $d_{ini}$
$\delta_{d_{res}}$ = Modification of the residual displacement $d_{res}$ due to the initial displacement $d_{ini}$

$\delta_U$ = deformation demand (chord rotation, $\theta$, or curvature, $\varphi$) at the end of the member, for the combination of the seismic action corresponding to the “Operational” performance level with the simultaneously acting gravity loads, from an elastic response analysis
$\delta_{m,m}$ = mean value of the deformation (chord rotation, $\theta$, or curvature, $\varphi$) capacity at member end
$\delta_{m,m}$ = is the mean-minus-standard deviation value of the deformation (chord rotation, $\theta$, or curvature, $\varphi$) capacity at member end
$\delta_{uk,0.05}$ = lower characteristic (5%-fractile) value of the deformation (chord rotation, $\theta$, or curvature, $\varphi$) at yielding at member end

$\varepsilon_{su,w}$ = ultimate strain of transverse reinforcement
$\varepsilon_y$ = yield strain of longitudinal reinforcement
$\varepsilon_{c0}$ = strain at maximum compression stress for unconfined concrete in the fibre model
$\varepsilon_{cr}$ = total strain in the biaxial concrete model
$\varepsilon_{tr}$ = residual strain after unloading in compression in the biaxial concrete model
$\varepsilon_{t1}$ = strain at maximum tensile stress in the fibre model
$\varepsilon_{tr}$ = crack opening strain in the fibre model
$\theta$ = chord rotation
$\theta_U$ = ultimate chord rotation
$\theta_y$ = yield chord rotation
$\theta^{\theta_{(x,y)}}$ = Reduced horizontal elastic rotations
$\theta^{\theta_{(x,y)}}$ = Reduced horizontal plastic rotations
$\mu$ = Coefficient of friction of the spherical sliding bearing
$M$ = Friction Coefficient
$\mu$ = shear retention factor in the biaxial concrete model
$v$ = axial load ratio
$\nu$ = Poisson coefficient of the soil
$\xi_0$ = is the compression zone depth at yielding (normalized to d)
$\rho_t$ = ratio of the tension reinforcement.
$\rho_c$ = ratio of the compression reinforcement.
$\rho_\theta$ = steel ratio of diagonal reinforcement in each diagonal direction
$\rho_v$ = transverse reinforcement ratio (minimum among the two transverse directions)
$\rho_w$ = ratio of the “web” reinforcement
$\xi_{tot}$ = total longitudinal reinforcement ratio
$\rho$ = Isotropic hardening variable
$\rho_c$ = maximum compression stress for unconfined concrete in the fibre model
$\sigma_{12}$ = shear stress in the biaxial concrete model
$\sigma_c$ = maximum compression stress for unconfined concrete in the fibre model
$\sigma_t$ = stress value defining the compression plateau for confined concrete in the fibre model
$\sigma_\psi$ = maximum tensile stress in the fibre model
$\bar{\tau}$ = Kinematic hardening array containing the current values of the hardening variables
$\|\bar{\tau}\|$ = Norm of the vector $\bar{\tau}$
$\bar{\tau}_{lim}$ = Kinematic hardening array containing the limits of the hardening variables
$\phi_u$ = ultimate curvature
$\phi_y$ = yield curvature
$\omega_c$, $\omega_t$ = mechanical reinforcement ratio of compression longitudinal reinforcement
$\omega_c'$ = mechanical reinforcement ratio of compression longitudinal reinforcement
$\omega_t'$ = mechanical reinforcement ratio of tension and “web” longitudinal reinforcement
$\omega_t$ = mechanical reinforcement ratio of tension and “web” longitudinal reinforcement
$\omega_t'$ = mechanical reinforcement ratio of tension longitudinal reinforcement
$\omega_c''$ = mechanical reinforcement ratio of compression longitudinal reinforcement